

Revealed Preference for Green Stocks: An Asset Demand Approach

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Abstract

This paper combines a traditional portfolio construction problem with demand estimation techniques to estimate the demand for green stocks of US institutional investors. The methodology presented innovates along two dimensions with respect to recent influential work on asset demand estimation. First, in our framework investors have heterogeneous portfolios not only through differential beliefs about future returns, but also because they place varying importance on the non-financial characteristics of the portfolios they construct. Second, by using a mixed logit demand specification, we can estimate asset demand that delivers more realistic substitution patterns across assets. Using data on the environmental performance of firms and quarterly stock holdings data from institutional investors, we estimate the demand for stocks accounting for environmental scores and return-related stock characteristics. We find that taste for green stocks fluctuates over time and by investor's assets under management. In a counterfactual exercise we study the equity price effects of a ban on green investing for pension funds; we find that a portfolio with the top brown stocks is estimated to have capital gains of 1.1% due to the policy, while a portfolio with the top green stocks is estimated to have capital losses of 1.6%.

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1 Introduction

The last decade has experienced a steady rise in sustainable investments, with global funds invested in sustainable funds reaching USD 2.7 trillion in the first quarter of 2023 (Morningstar (2023a)). This has been accompanied by growing interest in sustainable investment from asset managers, with 85% of them already implementing or planning to implement sustainable investing (Morgan Stanley (2022)), and a growing supply of sustainable funds available to investors (Morningstar (2023b)). Equity markets will play a fundamental role in the transition to an environmentally sustainable economy by providing incentives for listed firms to adopt cleaner technologies and practices. Understanding the demand of investors for green stocks and its consequences for equity prices is a key part of understanding the incentives of listed firms to align their business strategies with an environmentally sustainable economy.

This paper combines a traditional portfolio construction problem with demand estimation techniques to estimate the demand for green stocks of US institutional investors. To do this, it is necessary to have a model capable of dealing with the empirical fact that investors vary greatly in the portfolios they construct. One largely studied reason for portfolio heterogeneity is that investors construct different portfolios because they exhibit belief heterogeneity over the future returns of the assets they include in their portfolios. Another less studied reason for portfolio heterogeneity is that, even if investors have common beliefs, they assign varying importance to the characteristics of the portfolios they construct. That is, they exhibit taste heterogeneity over portfolio characteristics. For example, investors can tilt their portfolios toward environmentally sustainable, or green assets, for motives unrelated to future returns.

In this paper we present a framework to estimate demand for assets where both belief and taste heterogeneity play a role. Belief heterogeneity over future returns is codified via investor-specific conditional expectations, while taste heterogeneity allows investors to care about portfolio characteristics beyond those directly related to an expected return-versus-risk trade off. We estimate the demand for stocks accounting for environmental scores and return-related stock characteristics. Using the estimated demand, we conduct a counterfactual exercise to study the effects of a ban on green investing for pension funds, a policy discussed in the US Senate (Morgan (2023)), on equity prices and aggregate holdings.

This project innovates on the recent influential work by [Kojien and Yogo \(2019\)](#) (KY2019) that shows that an asset demand approach combined with a market clearing condition implies a valid asset pricing model. KY2019 rekindled a classic literature on modeling asset demand and advocate for applying industrial organization (IO) tools to asset pricing models. However, the microfoundations for asset demand presented in KY2019 are based exclusively on belief heterogeneity over future returns, not allowing for taste heterogeneity to influence investors' demand, and the substitution patterns between assets are restricted by the demand specification they use.

The methodological contributions of this paper are twofold. First, the microfoundations of the asset demand framework presented in this project also allow for taste heterogeneity in the portfolio construction problem in addition to belief heterogeneity. Allowing for taste heterogeneity in portfolio characteristics opens the door to studying investor behaviors that do not fit within into the traditional expected returns-versus-risk paradigm. Examples of such behaviors include: (i) investment strategies based on Environmental, Social, and corporate Governance (ESG) performance metrics of the companies, which take into account stock characteristics unrelated to returns. (ii) "Sin" stock dis-investing, where investors based on ethical considerations alone reduce their investments or completely avoid stocks that belong to so-called *sin* industries, like alcohol, tobacco, gambling, adult entertainment, or weapon manufacturing, based on ethical alone. (iii) The deletion of a stock from a stock index can mechanically change the demand for the stock if, for example, there are hedge funds or mutual funds designed to track the index, even if the fundamentals of the company remain unchanged.

The second contribution of this paper is to present and estimate a mixed-logit demand specification for the demand for stocks.¹ In this specification, heterogeneity is captured by investor-specific coefficients that are modeled as functions of investor demographics (see [Berry et al. \(1995\)](#) for the seminal application of this framework, [Berry and Haile \(2021\)](#), [Gandhi and Nevo \(2021\)](#), and [Conlon and Gortmaker \(2020\)](#) for modern practices on demand estimation for differentiated products). KY2019 derive a logit demand system for assets where price elasticities are proportional to portfolio shares. To see why this is a restrictive feature, imagine that the stocks for two companies have

¹The mixed logit demand is also referred to as random coefficients demand in the IO literature.

the same portfolio weights (market shares in this context), but these companies belong to different industries, for example technology and energy. These companies likely have very different fundamentals; however, under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks. The richer investor-level heterogeneity captured by mixed logit demand delivers flexible substitution patterns between assets, improving on the restrictive price elasticities of logit demand.

In this project we present an empirical application that studies the demand for stocks of institutional investors. To estimate this demand, we use data on US institutional investors and their holdings of US stocks combined with characteristics of the stocks they hold. To construct stock characteristics, we combine price and accounting data with information on the environmental performance of listed companies in the form of environmental scores (E-scores) from the MSCI rating agency. In this application we quantify the extent to which investors value the green metrics of the stocks they select for their portfolio while also considering returns-related characteristics. While it is not possible to distinguish whether each characteristic is included in the demand due to belief or taste heterogeneity using only holdings data, survey evidence supports the interpretation of environmental aspects as a taste characteristic.²

The estimates rely on quarterly data of the stock holdings of US institutional investors and the corresponding stock characteristics from 2001-Q1 to 2019-Q4, and we perform estimation in two-year windows. The first finding is that the revealed taste for green stocks fluctuates over time. Throughout the estimation sample we find a positive taste for green stocks as measured by a positive semi-elasticity for E-scores that is increasing in the second half of the sample. Moreover, in the period after the Great Recession (2007-Q4 to 2009-Q2) there is an increase in the range of values for the coefficient on E-scores, showing an increase in the heterogeneity in the sensitivity to green characteristics across investors. This suggests that after periods of economic downturn, some investors may care relatively more about the return-related characteristics of the stocks and relatively less about the environmental-friendliness of the companies underlying the stocks.

²Giglio, Maggiori, Stroebe, Tan, Utkus and Xu (2023) examine a survey of retail investors on the motives for Environmental Social and Governance (ESG) investing and find that generally investors expect ESG investments to underperform the market and that only 7% of investors in ESG assets were motivated by return expectations.

We also find that the coefficient corresponding to E-scores for a particular investor is a function of the investor's assets under management. We find that in the last ten years of the sample, institutions with more assets under management have on average a higher taste for green stocks. We repeat the estimation exercise under a logit demand specification where all investors share the same sensitivity to the green metrics. Such estimates exhibit much less variation.

In a counterfactual exercise, we use the estimated demand system for stocks to study the effects of a ban on green investing for pension funds on equity prices and aggregate holdings. On March 1st 2023, the US Senate passed a bill to prevent pension fund managers from basing investment decisions on factors like climate change (Morgan (2023)). President Biden vetoed the bill days later (Thomas (2023)), but various US State Legislatures have approved similar initiatives.³ In the counterfactual exercise this policy is implemented by making the demand for stocks of pension funds perfectly inelastic to the environmental performance of the stocks, so for these investor only return-related characteristics are taken into account in their demand for stocks.

Using the data and estimates for 2019-Q1, we find that stock with low E-scores, brown stocks, will benefit the most with higher counterfactual prices. A portfolio in the bottom quintile of green stocks is estimated to have an associated average price increase of 1.1% under the counterfactual. In contrast, the top quintile portfolio has an average price decrease of 1.6%. Results for the counterfactual exercise using a logit demand specification exhibit much smaller price changes due to the restrictive substitution patterns of logit demand.

Related literature. As mentioned above, this project is most closely related to Kojen and Yogo (2019). In that paper, the authors also propose a demand system approach to asset pricing and estimate a model that jointly explains asset prices and quantities. This project extends the framework from KY2019 in three ways. First, our model can accommodate taste heterogeneity allowing investors to consider stock characteristics beyond those related to returns when forming their portfolios. Second, a demand system with a mixed logit demand specification provides flexible substitution patterns between assets, improving on the restrictive elasticities of the logit demand model used in KY2019.

³The US House of Representatives later tried to override President Biden's veto but failed to secure the necessary votes for that measure (Foran and Wilson (2023)).

Third, KY2019 define the market as pools of investors, while I use a more natural market definition of the US stock market in a quarter. Estimation at the market level facilitates dealing with the endogeneity of prices and allows us to consider instrumental variables inspired by the IO literature.

More broadly, this paper fits into the literature that models asset demand from investors. Classic works include [Brainard and Tobin \(1968\)](#), [Rosen \(1974\)](#), and [Lucas \(1978\)](#). This literature has recently received renewed attention with the use of new stock holdings data and strategies to tackle endogeneity problems. Recent examples include KY2019 and [Kojien and Yogo \(2020\)](#). [Kojien and Yogo \(2020\)](#) builds on the tools presented in KY2019 to study a demand system for financial assets that includes currencies, bonds, and stocks across several countries. [Jiang, Richmond and Zhang \(2020\)](#) use a demand approach to portfolio construction to study global imbalances in net foreign assets across countries. Another example of a demand system approach is in [Han, Roussanov and Ruan \(2021\)](#), where the authors use KY2019's demand approach to quantify the impact of underperforming mutual funds on the overpricing of high-beta stocks. This project contributes to these recent papers by studying more than return characteristics as determinants of investors' demand curves, the use of a mixed logit demand specification, and the use of modern instrumental variables suggested by the IO literature.

A very recent but growing literature has studied how the environmental performance of stocks matters for equity holdings and prices. Theoretical approaches include [Pástor, Stambaugh and Taylor \(2021\)](#), where the authors develop a model where there are non-pecuniary benefits from investing in green assets, and such benefits lead to lower expected returns on green assets due to investors pushing up their prices. Empirical approaches include [Baker, Egan and Sarkar \(2022\)](#) that, using an asset demand approach, interpret the fees for ESG funds to find that investors are willing, on average, to pay 20 basis points more per annum to invest in a fund with an ESG mandate as compared to an otherwise identical mutual fund without an ESG mandate. [Pastor, Stambaugh and Taylor \(2023\)](#) also employ an asset demand approach to study the degree investors tilt their portfolios between green and brown stocks; they find that on aggregate institutional investors have become increasingly green, exhibiting a positive green tilt, while non-institutional investors have become browner, exhibiting a negative green tilt. [Kojien, Richmond and Yogo \(2023\)](#) use a demand system with stock characteristics related

to environmental performance to study the impact of climate-related induced shifts on equity prices. They study a counterfactual exercise where there is an increased sensitivity for green stocks and find this implies capital gains for passive investment institutions and capital losses for active investment institutions. With respect to these recent papers studying the demand of green stocks, this paper contributes by studying the effects of a ban on green investing on pension funds and employing the methodological contributions mentioned above.

The rest of this paper is organized as follows: section 2 presents the investor portfolio problem and how a demand for assets with a tractable logit functional form can be obtained from its solution. Section 3 presents how mixed logit demand can be estimated in the context of demand for stocks. Section 4 presents the empirical application that estimates the demand for green stocks from the institutional investors in the US. Section 5 shows the counterfactual exercise that studies the effects of a ban on green investing for pension funds. Finally, section 6 concludes and discusses future avenues of work.

2 Asset Demand with Taste Heterogeneity

In this section we show how a demand for assets with an empirically tractable logit functional form can be obtained from the solution of an traditional portfolio problem. In this set up we allow for investor belief and taste heterogeneity; we discuss how it relates to traditional portfolio problems in the asset pricing literature as well as recent key contributions that model asset demand. The exposition is divided in four subsections. The first one presents the investor portfolio problem and the second one its solution. Importantly, the third subsection presents the assumptions needed to obtain the empirically tractable demand for assets; and the fourth subsection presents the market clearing condition that pin downs equilibrium prices.

2.1 Investor Portfolio Problem

In this subsection we present the portfolio construction problem investors solve. The key assumption is that investors differ in their beliefs about future returns and assign varying importance to non-financial characteristics of the portfolios they construct.

To facilitate exposition for the remainder of the paper we consider the assets available to investors to be stocks, but the framework in this section applies to asset classes other than stocks, and of course to combinations of assets from different classes. Let $t = 1, \dots, T$ denote the stock market in given period. In our application the market definition corresponds to the US stock market at a quarter t . In each of these markets there are I_t investors indexed by $i = 1, \dots, I_t$, and each investor i has to allocate A_{it} dollars of assets under management (AUM) in market t among J_t available stocks and an outside option. Stocks are treated as differentiated investment products that are demanded by investors. Let $j = 1, \dots, J_t$ index one of the J_t available stocks and $j = 0$ denote the outside option⁴. In our context the outside option denotes the possibility that investors allocate a fraction of their AUM into none of the stocks in J_t .

Let R_{t+1} denote a J_t -vector of gross returns between t and $t + 1$ for the stocks available in period t ; similarly R_{t+1}^0 denotes the gross return on the outside asset. For each stock j the gross return between t and $t + 1$ is computed as $R_{t+1,j} = \frac{V_{t+1,j}}{P_{t,j}}$, where $V_{t+1,j}$ is the payoff per share⁵ of stock j in $t + 1$, and $P_{t,j}$ is the price per share of stock j in t .

Each investor solves a two-period problem between the current period (t) and the next period ($t + 1$) where they construct a portfolio by choosing portfolio weights w_{it} (a J_t -dim vector), such that

$$\max_{w_{it}} E_{it}[\log(A_{i,t+1})] + a'_i C'_t w_{it} \quad (1)$$

$$\text{s.t. } A_{i,t+1} = A_{it} \left[R_{t+1}^0 + w'_{it} [R_{t+1} - R_{t+1}^0 \mathbf{1}] \right] \quad (2)$$

$$w_{it} \geq 0; \quad \mathbf{1}' w_{it} < 1 \quad (3)$$

In this problem investors differentiate portfolios according to the characteristics they offer. We separate the characteristics of the portfolio into tomorrow's terminal wealth $A_{i,t+1}$, and additional characteristics C_t of the stocks that composed portfolio w_{it} . The part of utility that comes from tomorrow's dollar value for the portfolio enters through a log utility, while the current value for investor i of other portfolio characteristics enters

⁴When there is no possibility of confusion, we use J_t to denote the set of inside goods and the cardinality of the set itself, that is $J_t = \{1, 2, \dots, |J_t|\}$.

⁵In many contexts $V_{t+1,j}$ is divided as sum of the price per stock of stock j in $t + 1$, $P_{t+1,j}$ plus the dividends per stock in $t + 1$, $D_{t+1,j}$; however, in our setting investors only care about the overall future payoff $V_{t+1,j}$, and not whether it was generated by capital gains or dividends.

with linear weights a_i , an investor-specific K_C -vector. The values of a_i capture investor preferences over the characteristics included in C_t . The log specification in the utility for tomorrow's portfolio wealth follows KY2019 and a long tradition that dates back to Samuelson (1969)⁶. If the entries of a_i are set to zero, taste heterogeneity is irrelevant and we are in a context where only pecuniary factors matter for portfolio construction.

The matrix C_t denotes a $J_T \times K_C$ matrix where row j contains a K_C -vector c_{jt} of characteristics for stock j that are relevant for the profile of portfolio w_{it} . $E_{it}[\cdot]$ denotes the conditional expectation for investor i at time t , that is $E_{it}[\cdot] \equiv E[\cdot | \mathcal{I}_{it}]$, where \mathcal{I}_{it} denotes the information set of investor i at time t . In this model it is assumed that investors do not learn from the actions of others investors; this assumption is commonly referred in the literature as *investors agree to disagree*. The first constraint in (2) denotes the evolution of the portfolio's wealth by choosing portfolio w_{it} and the constraints in (3) impose short-sale restrictions⁷ and that all wealth is invested in either stocks or the outside option.

Including stock characteristics into the value of selecting a portfolio w_{it} may be relevant to empirically capture investment decisions that do not entirely fit into an expected return-versus-risk investment paradigm. Examples of this type of investment behavior include: (i) green stock investing where investor value the environmental performance of the stocks they include in their portfolios, (ii) more generally in investment strategies based on environmental, social, and corporate governance (ESG) metrics of the companies that not only take into account returns and wealth accumulation when selecting stocks to invest. (iii) "Sin" stock dis-investing, where investor avoid including stocks that belong to a so-called "sin" industry like tobacco, alcohol, gambling, adult entertainment or guns; despite the returns these stocks may offer. (iv) Finally, addition or deletion of a stock into an stock index (e.g. S&P 500, Russell 1000 or The Dow Jones Industrial Average) can mechanically induce demand for the stock, for example from hedgefunds and mutual funds designed to track the index, despite stock fundamentals may remain unchanged. This modeling choice generalizes the setup in KY2019, where next's period portfolio wealth, $A_{i,t+1}$, is the only relevant characteristic to construct a portfolio.

Notice that the problem in (1) accommodate two sources of heterogeneity. First,

⁶In a multi period setup, assuming Log utility collapses the portfolio problem into a two-period problem, as in our setup.

⁷For a paper that relaxes short sale constraints see Tian (2022).

it captures belief heterogeneity over future returns, which is codified via differential information on the expectations operator, $E_{it}[\cdot]$. Under homogenous beliefs it would be the case that $E_{it}[\cdot] = E_t[\cdot]$ for all i . And second, it captures taste heterogeneity over stock characteristics via the weights a_i , if all investors value portfolio characteristics equally then $a_i = a$ for all i . Together, the objective function in (1) accommodates these two channels for heterogeneity.

2.2 Optimal Portfolio Weights

This subsection presents the solution to the portfolio construction problem. The approximate solution for positive portfolio weights has a traditional form that depends on the first two moments of expected returns but also takes into account investor preferences over non-financial characteristics.

In order to provide the solution to the investor's problem we introduce some notation. Denote by r_{t+1}^x the vector of excess log returns, $r_{t+1}^x = \log(R_{t+1}) - \log(R_{t+1}^0)1$. Moreover, let $\tilde{\Sigma}_{it}$, a $J_t \times J_t$ matrix, denote the variance-covariance matrix

$$\tilde{\Sigma}_{it} = E_{it} \left[(r_{t+1}^x - E_{it}[r_{t+1}^x]) (r_{t+1}^x - E_{it}[r_{t+1}^x])' \right],$$

and let $\tilde{\mu}_{it}$, a J_t -vector of conditional expectations adjusted by variance:

$$\tilde{\mu}_{it} = E_{it}[r_{t+1}^x] + \frac{\tilde{\sigma}_{it}^2}{2},$$

where $\tilde{\sigma}_{it}^2$ is a vector of the diagonal elements of $\tilde{\Sigma}_{it}$. Furthermore, without loss of generality, partition the asset space between the J_t^1 assets with positive weights on the investor's problem, that is those assets for which short sale constraints are not binding so we can rewrite $\tilde{\Sigma}_{it}$ and $\tilde{\mu}_{it}$ as

$$\tilde{\mu}_{it} = \begin{pmatrix} \mu_{it} \\ \mu_{it}^{(2)} \end{pmatrix}, \quad \tilde{\Sigma}_{it} = \begin{pmatrix} \Sigma_{it} & \Sigma_{it}^{(1,2)} \\ \Sigma_{it}^{(2,1)} & \Sigma_{it}^{(2,2)} \end{pmatrix}$$

where μ_{it} is a J_t^1 -vector and Σ_{it} a $J_t^1 \times J_t^1$ matrix, both corresponding to the assets with positive weights. The following proposition parallels Lemma 1 in the KY2019 framework in characterizing the solution for the optimal portfolio but accounts for the extra term

that allows for taste heterogeneity

Proposition 1. *The solution to the investor problem in (1)-(3), w_{it}^* , is characterized by the Euler equation*

$$E_{it} \left[\left(\frac{A_{it+1}}{A_{it}} \right)^{-1} R_{t+1} \right] = 1 - \left(I - 1w_{it}^* \right) (\Lambda_{it} - \lambda_{it}1 + C_t a_i) \quad (4)$$

where Λ_{it} and λ_{it} are Lagrange multipliers on (2) and (3) respectively. Moreover, the positive optimal portfolio weights can be approximated (over a short period horizon) as

$$w_{it}^* \approx \Sigma_{it}^{-1} (\mu_{it} - \lambda_{it}1 + C_t a_{it}) \quad (5)$$

The proof for Proposition 1 is shown in Appendix A. The Euler equation in (4) generalizes the set up in KY2019, since the case with no taste heterogeneity, $a_i = 0$, results in the Euler equation presented in KY2019's model⁸. Furthermore, as in KY2019, if investors do not face short-sale constraints ($\Lambda_{it} = 0$ and $\lambda_{it} = 0$) and have homogeneous beliefs ($E_{it}[\cdot] = E_t[\cdot]$ for all i), then the Euler equation becomes

$$E_t \left[\left(\frac{A_{it+1}}{A_{it}} \right)^{-1} R_{t+1} \right] = 1$$

which is a moment condition commonly tested in the literature on consumption-based asset pricing. The message in the second part of proposition 1, is that investor i 's demand for stocks, given by portfolio weights w_{it}^* , is determined by three components: uncertainty around next period returns (Σ_{it}), expected returns (via μ_{it}) and taste sensitivity ($C_t a_{it}$). All else equal, if investor i has more uncertainty around next period returns for some assets then the portfolio weights on those assets will be relative smaller. Similarly, keeping Σ_{it} fixed, stock holdings are increasing on expected returns. More generally, in our setup stock holdings vary according to the additional value stocks contribute the investors' utility derived from portfolio characteristics. If an stocks j contribute positively to a portfolio characteristics k valued positively by investor i , $(a_{it,k} c_{jt,k}) \geq 0$, then increasing such characteristic for stock j will imply that investor i holds relative more stocks of j .

⁸Lemma 1 in Kojien and Yogo (2019), page 1481.

2.3 An empirically tractable demand for stocks

Despite the fact that equation (5) provides a clear intuition of the determinants of the optimal positive portfolio weights, it is not very tractable empirically⁹. This subsection presents the set of assumptions that are needed to derive a form of equation (5) that uses characteristics of the stocks in a logit form to provide an empirical tractable function for the portfolio weights.

To compute the conditional moments that determine portfolio weights in equation 5, namely Σ_{it} and μ_{it} ; we need to be explicit about how next period's excess log returns are modeled and how information varies across investors. Recall that by definition $R_{t+1,j} = V_{t+1,j}/P_{t,j}$, where $V_{t+1,j}$ stands for next's period payoff for stocks j and $P_{t,j}$ is the equilibrium price per share. Similarly, $R_{t+1,0} = V_{t+1}^0/P_0$; and without loss of generality we normalize the price of the outside good to one, $P_0 = 1$. If we take logs we obtain that the vector of excess log returns is given by

$$\begin{aligned} r_{t+1}^x &= \log(V_{t+1}) - \log(V_{t+1}^0)\mathbf{1} - \log(P_t) \\ &= v_{t+1} - v_{t+1}^0\mathbf{1} - p_t \\ &:= v_{t+1}^x - p_t \end{aligned}$$

Given that the excess payoff in $t + 1$ is unknown at period t , we assume investors have a prior in period t about its value such that V_{t+1} and V_{t+1}^0 follow a lognormal distribution:

Assumption 1. *Distribution of next period's payoff*

The J_t -vector of next period's payoff, V_{t+1} , follows the lognormal distribution

$$V_{t+1} \sim \text{lognormal}(\mu_{vt}, \Sigma_{vt}),$$

where μ_{vt} is a J_t -vector and Σ_{vt} a $J_t \times J_t$ matrix. Moreover, the outside option next period's payoff also follows a lognormal distribution

$$V_{t+1}^0 \sim \text{lognormal}(\mu_{vt}^0, \Sigma_{vt}^0),$$

⁹It requires obtaining the first two investor-specific conditional moments over excess log returns; which is composed of a large cross section of stock returns.

where μ_{vt}^0 and Σ_{vt}^0 are scalars. The values of $\mu_{vt}, \mu_{vt}^0, \Sigma_{vt}, \Sigma_{vt}^0$ are common knowledge to investors.

Since next period's payoffs are bounded from below by zero, the log normality assumptions is an appropriate modeling choice and it is a traditional assumption in asset pricing. Assumption 1 implies that the excess log returns can be written as

$$r_{t+1}^x = \mu_{vt} - \mu_{vt}^0 \mathbf{1} - p_t + e_v \quad (6)$$

where

$$\begin{aligned} e_v &\sim N(0, \Sigma_{xt}) \\ \Sigma_{xt} &= E_t[(r_{t+1}^x - \mu_{xt})(r_{t+1}^x - \mu_{xt})'] \\ \mu_{xt} &= \mu_{vt} - \mu_{vt}^0 \mathbf{1} - p_t, \end{aligned}$$

and the (j, k) -entry of Σ_x is given by:

$$\Sigma_{xt,jk} = \Sigma_{vt,jk} + \Sigma_{vt}^0 - \text{cov}(V_{t+1,j}, V_{t+1}^0) - \text{cov}(V_{t+1,k}, V_{t+1}^0).$$

Notice that the value of excess log returns depends on the vector of prices, p_t , which need to be pinned down in equilibrium. We also assume that equilibrium prices are observed by all investors. Then conditional on public information, excess log returns can be viewed as having a distribution inherited from the distribution assumed for next period excess payoffs¹⁰. In this case $r_{t+1}^x \sim N(\mu_{xt}, \Sigma_{xt})$, but both μ_{xt} and Σ_{xt} depend on p_t which is endogenous to the model. The next assumption states a factorization for the matrix Σ_{xt} .

Assumption 2. *Factorization for the matrix Σ_{xt}*

The matrix Σ_{xt} admits the representation

$$\Sigma_{xt} = \Gamma_{xt} \Gamma_{xt}' + \sigma_e^2 I. \quad (7)$$

where Γ_{xt} is a J_t -vector of factor loadings and σ_e^2 is common variance across stocks.

This assumption is consistent with a factor structure for the vector of log excess

¹⁰Unfortunately, the setup in KY2019 is not explicit about to what extent returns are endogenous or exogenous to their setup, we believe that being explicit about this and modeling how information is different across investors contribute to the clarity of the microfoundations of asset demand.

returns where r_{t+1}^x admits the following single factor representation:

$$r_{t+1}^x = \mu_{xt} + \Gamma_{xt}F_{t+1} + e_{t+1},$$

the single factor F_{t+1} is distributed as $N(0, 1)$ and $e_{t+1} \sim N(0, \sigma_\varepsilon^2 I)$, with e_{t+1} independent of F_{t+1} ¹¹.

In order to compute moments that depend on the information set each investor have, we model how information varies across investors. We adapt a traditional setup from the asymmetric information literature in asset pricing (see for example Grossman (1976)).

Assumption 3. Information Technology

Each investor i receives a signal about next period's excess log returns, r_{t+1}^x , denote by s_{it} , a J_t -vector and given by

$$s_{it} = \alpha_i r_{t+1}^x + \varepsilon_{it} \quad (8)$$

where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2 I)$ and is independent of r_{t+1}^x . The scalar constant α_i is privately known to investor i .

In this set up, each investors knows s_{it} and α_i but does not know the value of ε_{it} that produced s_{it} so they cannot back out immediately the value of r_{t+1}^x and has to update her expectations over r_{t+1}^x by conditioning on the information set $\mathcal{I}_{it} = \{s_{it}, \alpha_i, \mu_{xt}, \Sigma_{xt}, \sigma_\varepsilon^2\}$. Notice that $s_{it} | (r_{t+1}^x, \alpha_i) \sim N(\alpha_i r_{t+1}^x, \sigma_\varepsilon^2 I)$ and $r_{t+1}^x \sim N(\mu_{xt}, \Sigma_{xt})$ so by Bayes theorem we can compute the distribution of $r_{t+1}^x | (s_{it}, \alpha_i)$ which corresponds to the distribution of $r_{t+1}^x | \mathcal{I}_{it}$. The posterior distribution is given by

$$r_{t+1}^x | \mathcal{I}_{it} \sim N(\mu_{r|s_i}, \Sigma_{r|s_i}) \quad (9)$$

$$\text{with } \Sigma_{r|s_i} = \left[\alpha_i^2 (\sigma_\varepsilon^2)^{-1} I + \Sigma_{xt}^{-1} \right]^{-1} \quad (10)$$

$$\mu_{r|s_i} = \Sigma_{r|s_i} \left[(\sigma_\varepsilon^2)^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt} \right] \quad (11)$$

Details on the derivation of the posterior moments is shown in appendix A.

The next proposition shows that under assumptions 1 and 2 we can obtain a convenient decomposition of the investor-specific conditional matrix Σ_{it}

¹¹Alternatively, the factorization in Assumption 2 can be obtained by assuming a factor structure on the variance matrix of future payoffs Σ_{vt} . In this case notice that $\sigma_\varepsilon^2 = \Sigma_{vt}^0$.

Proposition 2. Under assumptions 1 to 3, the investor-specific conditional matrix Σ_{it} can be written as $\Sigma_{it} = \Gamma_{it}\Gamma'_{it} + \iota_{it}I$ where Γ_{it} is an J_t -vector and ι_{it} a scalar, both investor-specific.

Proof of Proposition 2 is shown in appendix A. Notably, this result is an assumption in KY2019's model¹²; in our set up we are able to obtain the investor-specific decomposition of Σ_{it} by relying on weaker assumptions. The next assumption relates the first two moment of the vector of excess log returns with individual stock characteristics.

Assumption 4. Return-related Stock Characteristics

Each entry of the J_t -vectors μ_{xt} and Γ_{xt} can be expressed as a polynomial of order M over a K_x -vector of return-related stock characteristics x_{jt} , including price p_{jt} ; that is

$$\mu_{xt,j} = y'_{jt}\Phi_\mu + \phi_\mu \quad (12)$$

$$\Gamma_{xt,j} = y'_{jt}\Phi_\Gamma + \phi_\Gamma \quad (13)$$

where Φ_μ and Φ_Γ are matrices of coefficients, ϕ_μ and ϕ_Γ scalars and y_{jt} is K_y -vector with $K_y = \sum_{m=1}^M K_x^m$ and

$$y_{jt} = \begin{pmatrix} x_{jt} \\ x_{jt} \otimes x_{jt} \\ x_{jt} \otimes x_{jt} \otimes x_{jt} \\ \vdots \end{pmatrix},$$

and \otimes stands for the Kronecker product.

The motivation for the previous assumption is twofold. First, modeling the mean and covariance matrices of asset returns as functions of assets characteristics is more empirically tractable than estimating the mean and covariance matrices of asset returns directly from past observations (see for example Brandt et al. (2009)).¹³ Second, KY2019 show the empirical the relevance of characteristics-based demand (their appendix B), by showing that expected returns and factor loadings are well captured by a few asset characteristics, and that this approach better estimates the mean-variance portfolio compared to a benchmark that uses sample estimates of the first two moments of returns.

¹²The first part of Assumption 1 in Kojien and Yogo (2019), p. 1483.

¹³Since the mean and covariance matrices of asset returns are hard to estimate and very likely time-varying, this literature moved from estimation using historical samples of returns to using functions that map assets characteristics to their returns, and hence the first two moments of returns. The rationale is that asset characteristics are more stably related to expected returns than company names in a large cross section.

Notice that Assumption 4 introduces a second set of asset's characteristics. On one hand the vector x_{jt} of stock characteristics, including prices p_{jt} , that are relevant for the conditional expected returns (μ_x) and factor loadings (Γ_x), we called them *return-related characteristics*. On the other hand c_{jt} , that denotes the stock characteristics that are relevant the portfolio characteristics valued by investors, we called them *taste characteristics*. Empirically, this suggest to include observable stock characteristics known to be relevant for the cross section of return into x_{jt} and those relevant for investment decisions but not directly related to stock returns into c_{jt} .

The following proposition uses all previous assumptions and results to characterize optimal portfolio weights as polynomials on asset characteristics with investor specific coefficients, and then obtain the empirically tractable logit form that relates portfolio holdings with stock characteristics.

Proposition 3. *Under Assumptions 1 to 4:*

- (i) *The j -th entry of the vectors Γ_{it} and μ_{it} can be written as polynomial functions on x_{jt} with investor-specific coefficients.*
- (ii) *The optimal portfolio weights for each asset j with positive weight, $w_{it,j}$ can be written as polynomial function on asset characteristics (x_{jt}, c_{jt}) with investor specific coefficients:*

$$w_{it,j} = \tilde{y}'_{jt} \Phi_{w,i} + \phi_{w,i}, \quad (14)$$

where $\Phi_{w,i}$ is vector of coefficients, $\phi_{w,i}$ is a scalar, and \tilde{y}_{jt} is a $K_{\tilde{y}}$ -vector with $K_{\tilde{y}} = \sum_{m=1}^{2M} (K_X + K_C)^m$, and

$$\tilde{y}_{jt} = \begin{pmatrix} \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \vdots \end{pmatrix},$$

and $\tilde{x}_{jt} = (x'_{jt} \quad c'_{jt})'$ a $(K_X + K_C)$ -vector.

- (iii) *Moreover if the polynomial order M goes to infinity, $M \rightarrow \infty$, then a restriction of parameters implies that the optimal portfolio weights for investor i can be written as:*

$$w_{it,j} = \frac{\exp \left(x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \zeta_{jt} \right)}{1 + \sum_{j=1}^{J_t} \exp \left(x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \zeta_{jt} \right)}, \quad (15)$$

and the portfolio weight for the outside option is given by

$$w_{it,0} = \frac{1}{1 + \sum_{j=1}^{J_t} \exp \left(x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \zeta_{jt} \right)}, \quad (16)$$

where b_{it} and γ_{it} are investor-specific coefficients on observed returns-related characteristics x_{jt} , that include price p_{jt} , and taste characteristics c_{jt} . The term ζ_{jt} represents an index of unobserved (by the econometrician) return-related characteristics.

It is possible that there are asset characteristics unobserved by the econometrician but relevant for investor's portfolios. Without loss of generality we can assume those unobserved characteristics are summarized in an index ζ_{jt} and that this index is a characteristic included in x_{jt} as in part (iii) of proposition 3. In the previous the coefficient on the unobserved characteristic ζ_{jt} is normalized to 1 and the portfolio value of the outside option is normalized to 1. Part (i) of this proposition is an assumption in KY2019's framework, as in proposition 2, we are able to obtain this result by relying on weaker assumptions. Proof of proposition 3 is presented in Appendix A¹⁴.

This proposition tell us that the optimal weight on asset j for investor i in market t is directly explained by the characteristics of asset j , investor-specific coefficients (b_{it}, γ_{it}) , and the value that investor i assigns to j relative to all the other assets available in J_t .

2.4 Market Clearing Condition

The portfolio weights in (15) represent the asset demand curves for investors taking stock characteristics, including price as given. In this subsection we pair the demand system with a supply side to obtain a market clearing conditions that pin downs equilibrium prices. The key assumption is that the number of shares outstanding is fixed in the short run. In the empirical application of this paper we work with the stock market at a quarterly frequency so we consider this assumption reasonable.

¹⁴Once part (i) is proved, the proof of parts (ii) and (iii) proceed similarly as in proposition 1 of KY2019.

Let S_{jt} denote the number of shares outstanding for stock j in market t ; in the short run this is assumed to be a fixed number which can be interpreted as an inelastic supply of the stock. If we multiply S_{jt} by P_{jt} , the price per share of j in t , we obtain the market equity for stock j , denote by ME_{jt} .

Since each stock of j should be held by an investor in the market, then the following market clearing should hold:

$$ME_{jt} = \sum_{i=1}^I A_{it} w_{it,j}. \quad (17)$$

This market clearing condition says that all the money invested by investors in stock j , should equal the market equity of the stock. This condition is a re-statement in dollar value, instead of quantities, that prices in equilibrium are such that in the aggregate, supply equals demand:

$$S_{jt} = \frac{\sum_{i=1}^I A_{it} w_{it,j}}{P_{jt}}.$$

Notice that the right hand side of (17) depends on prices, since p_{jt} enters demand via de the return-related characteristic x_{jt} . As noted in KY2019, the market clearing condition implies a fixed point equation in p_{jt} . Let p_t be the J_t -vector of log prices for the stocks in market t and define $f : \mathbb{R}^{J_t} \rightarrow \mathbb{R}^{J_t}$ as:

$$f(p_t) = \log \left(\sum_{i=1}^I A_{it} w_{it,j}(p_t) \right) - \log(S_{jt}), \quad (18)$$

so using equation 18 we can solve for the equilibrium prices, by looking for p_t^* such that $p_t^* = f(p_t^*)$. KY2019 state the conditions for an unique fixed point to exist and present an algorithm for the determining such fixed point.

3 Demand Specification and Estimation

3.1 Demand Specification

Using data on portfolio holdings, $\{w_{it,j}\}$ and stock characteristics $\{x_{jt}, c_{jt}\}$ the goal is to estimate the asset demand coefficients defined in (15). This project uses a mixed logit demand specification (Berry et al. (1995)), also known as a random coefficients demand (RC) specification, where the investor-specific coefficients in b_{it} and γ_{it} follow the structure:

$$b_{it} = b_0 + \Pi_b d_{it} + \Sigma_b^{1/2} v_{b,it} \quad (19)$$

$$\gamma_{it} = \gamma_0 + \Pi_\gamma d_{it} + \Sigma_\gamma^{1/2} v_{\gamma,it}, \quad (20)$$

where $b_{0,k}$ is a component of $b_{it,k}$, common to all investors; d_{it} denotes an L -vector of observable investor demographics that are relevant to characterize $b_{it,k}$, along with the corresponding coefficients in Π_b , a $K_X \times L$ matrix. The term $v_{b,it}$ is a K_X -vector of investor-specific taste shocks that are scaled by common variance covariance coefficients, Σ_b . The $v_{b,it}$ can also be interpreted as unobservable (by the econometrician) investor demographics relevant for b_{it} . Analogous interpretations apply to γ_0 , Π_γ , Σ_γ and $v_{\gamma,it}$.

If we restrict to zero the matrices Π_b , Π_γ , Σ_b and Σ_γ , we have that $b_{it} = b_0$ and $\gamma_{it} = \gamma_0$ for all i . This is the highly studied logit demand. In this specification all investors have the same coefficients and is the one used during estimation in KY2019¹⁵. Logit demand is highly tractable but delivers restrictive substitution patterns. This is because in Logit demand price elasticities are determined by market shares. Leading authors in the demand estimation literature call this “*a bug not a feature*” (Berry and Haile (2021), pg. 19). The limitations imply that for small portfolio weights, own-price elasticities are approximately proportional to the coefficient corresponding to price. Moreover, two stocks with similar portfolio weights would react identically to the price change of any other stock¹⁶.

¹⁵In KY2019, a logit demand is estimated investor by investor, so they obtain a set of estimated coefficients by investor. However, at the investor level the substitution patters between stocks are those of logit demand.

¹⁶Berry and Haile (2021) develop their argument further: “These restrictions do not come from economics but from assumptions chosen for simplicity or analytical convenience. Models must, of course, abstract from reality, and finite samples require appropriate parsimony. But good modeling and approximation methods should aim to avoid strong a priori restrictions on the very quantities of interest unless those restrictions can be defended as natural economic assumptions.” (pg. 19).

Imagine two stocks that have the same portfolio weights, but the companies belong to different sectors, for example technology and energy. It is easy to imagine these stocks would respond to different fundamentals, yet under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks.

The restrictive substitution patterns of logit demand are a manifestation of the Independence of Irrelevant Alternatives (IIA) assumption (see, e.g. Arrow (1951), Ray (1973), McFadden (1974)), that states that the relative likelihood of choosing between two options will not change on whether a third alternative is present. Because of IIA, logit demand will fail to capture substitution patterns between close substitutes, a feature that has been widely studied in the industrial organization literature. Extensions of logit demand like the nested logit demand (Cardell (1997)) and the mixed logit demand relax the IIA assumption. In nested logit the product choice is sequential, first individuals choose a product nest, and then they choose a product within the nest. Models of this type can be easily accommodated in a mixed logit demand by including the nest-defining characteristics as one of the characteristics in the demand specification.

Price Elasticities. Let $Q_{it,j} = \frac{A_{it}w_{it,j}}{P_{t,j}}$ denote the number of shares of stock j held by investor i in market t . The elasticity of stock j holdings when the price of stock k changes, denoted by $\eta_{it,jk}$ is given by:

$$\begin{aligned}\eta_{it,jk} &= \frac{\partial Q_{it,j}}{\partial P_{t,k}} \frac{P_{t,k}}{Q_{it,j}} = \frac{\partial \log(Q_{it,j})}{\partial \log(P_{t,k})} \\ &= \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,j})} - 1\{j = k\} := e_{it,jk} - 1\{j = k\}.\end{aligned}$$

The term $e_{it,jk}$ is also an elasticity but with respect to portfolio weights and hence depends on the demand specification. As mentioned above log prices, p_{jt} is one of the stock characteristics included in x_{jt} . Without loss of generality, let $k = 1$ denote index for the coefficient corresponding to log prices, $b_{it,1}$. Following this notation, the term $e_{it,jk}$ under logit demand ($b_{it,1} = b_{0,1}$ for all i) is given by:

$$e_{it,jk}^{Logit} = \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,k})} = b_{0,1}(1\{j = k\} - w_{it,k}).$$

The corresponding term for RC demand, $e_{it,jk}$, is given by

$$e_{it,jk}^{RC} = \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,k})} = \left(b_{0,1} + \sum_{\ell=1}^L \pi_{1,\ell} d_{\ell it} + \sigma_{t,1} v_{i,1} \right) (1\{j = k\} - w_{it,k}),$$

this term, $e_{it,jk}$, represents the price elasticity given the idiosyncratic preferences of investors as capture by the price coefficient $b_{it,1}$. However, this expression is not feasible to compute given that the taste shocks, $v_{i,1}$, are not observed. To compute the price elasticity conditional on observable data is necessary to integrate over the distribution of the taste shocks. If F_v denotes the distribution of v_{it} we can numerically integrate out its role on portfolio holdings using F_v and computing $e_{it,jk}^{RC}$ using $E_{F_v} [w_{it,j}]$:

$$\begin{aligned} e_{it,jk}^{RC} &= \frac{\partial \log(E_{F_v}[w_{it,j}])}{\partial \log(P_{t,k})} = \frac{\partial E_{F_v}[w_{it,j}]}{\partial P_{t,k}} \frac{P_{t,k}}{E_{F_v}[w_{it,j}]} \\ &= \frac{1}{E_{F_v}[w_{it,j}]} \int \left[\left(b_{0,1} + \sum_{\ell=1}^L \pi_{1,\ell} d_{\ell it} + \sigma_{t,1} v_{i,1} \right) \tilde{w}_{it,j}(v_i) (1\{j = k\} - \tilde{w}_{it,k}(v_i)) \right] dF(v_i). \end{aligned}$$

Comparing the terms $e_{it,jk}^{Logit}$ and $e_{it,jk}^{RC}$ shows, as mentioned at the beginning of this section, that a logit demand specification is limited in the substitutions patterns it can accommodate when stock prices change. RC demand, on the other hand, by employing a richer structure on the parameters, can deliver more flexible substitution patterns.

3.2 Estimation

The relevant equations for estimation are the mapping between asset characteristics and asset holdings, equation (15), paired with the random coefficients specifications that relate investor-specific coefficients with their demographics in (19) and (20). For convenience we present again these equations:

$$\begin{aligned} w_{it,j} &= \frac{\exp(x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \zeta_{jt})}{1 + \sum_{j=1}^J \exp(x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \zeta_{jt})} \\ b_{it} &= b_0 + \Pi_b d_{it} + \Sigma_b^{1/2} v_{b,it} \\ \gamma_{it} &= \gamma_0 + \Pi_\gamma d_{it} + \Sigma_\gamma^{1/2} v_{\gamma,it}. \end{aligned}$$

The data is composed of asset holdings, investor demographics and assets under management $\{w_{it}, d_{it}, A_{it}\}_{i=1, \dots, I_t}$ and asset characteristics $\{x_{jt}, c_{jt}\}_{j=1, \dots, J_t}$. The parameters to estimate are the components that form the coefficients corresponding to return and taste characteristics; namely $\{b_0, \Pi_b, \Sigma_b\}$ for the return characteristics x_{jt} and $\{\gamma_0, \Pi_\gamma, \Sigma_\gamma\}$ for the taste characteristics c_{jt} .

For exposition convenience of the estimation steps, let's rewrite observed asset characteristics into the $K = (K_X + K_C)$ vector $X_{jt} = (x'_{jt}, c'_{jt})'$, and accordingly define the coefficients vector $\beta_{it} = (b'_{it}, \gamma'_{it})'$ such that

$$\beta_{it} = \beta_0 + \Pi d_{it} + \Sigma^{1/2} v_{it} \quad (21)$$

where $\beta_0 = (b'_0, \gamma'_0)'$; Π is a $K \times L$ matrix with the demographics coefficients (Π_b, Π_γ) ; $v_{it} = (v_{b,it}, v_{\gamma,it})'$ a K -vector, and $\Sigma^{1/2}$ a $K \times K$ matrix composed of $(\Sigma_b^{1/2}, \Sigma_\gamma^{1/2})$. With this notation the parameters to estimate can be denoted as $\theta := (\beta_0, \Pi, \Sigma)$. In the IO literature for demand estimation $\theta_1 = \beta_0$ is commonly referred as "linear parameters" and $\theta_2 := (\Pi, \Sigma)$ as "non-linear parameters", due to the way these parameter enter the estimation procedure.

With this notation we can write the exponents in the expression for $w_{it,j}$ as

$$\begin{aligned} x'_{jt} b_{it} + c'_{jt} \gamma_{it} + \xi_{jt} &= X'_{jt} [\beta_0 + \Pi d_{it} + \Sigma v_{it}] + \xi_{jt} \\ &= \underbrace{X'_{jt} \beta_0 + \xi_{jt}}_{:=\delta_{jt}} + \underbrace{X'_{jt} [\Pi d_{it} + \Sigma v_{it}]}_{:=h_{ijt}(\theta_2, v_{it})} \\ &= \delta_{jt} + h_{ijt}(\theta_2, v_{it}). \end{aligned}$$

The term δ_{jt} is referred as the "mean utility" for option j in market t , as it is a common component for all investors; while the term $h_{ijt}(\theta_2, v_{it})$ captures investor-specific heterogeneity. Furthermore, we can write $w_{it,j}$ as

$$w_{it,j} = \frac{\exp(\delta_{jt} + h_{ijt}(\theta_2, v_{it}))}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt} + h_{ijt}(\theta_2, v_{it}))} \quad (22)$$

The next step is to obtain aggregate market shares for each stock. In this project we perform estimation at the market level. This is motivated because market-level estima-

tion facilitates dealing with the endogeneity of prices, as discussed below, so we can use instrumental variables for prices that have been suggested in the IO literature of demand estimation. This choice is also consistent with the market definition presented in the microfoundations, namely we consider a market to be US stock market in a given a quarter.

We can construct aggregate market shares from the market clearing condition (17). Let ME_{0t} denote the aggregate investment in the outside option: $ME_{0t} = \sum_{i=1}^{I_t} A_{it}w_{i0t}$; and denote by ME_t denote the aggregate value of the market in t : $ME_t = \sum_{j=0}^{J_t} ME_{jt}$. By the market clearing condition it has to be the case that the aggregate value of the market is equal to the aggregate assets under management across investors, so we have that $ME_t = \sum_{i=1}^{I_t} A_{it} := A_t$. If we divide the market clearing condition (17) by ME_t (or A_t equivalently) we obtain that

$$\begin{aligned} ME_{jt} &= \sum_{i=1}^{I_t} A_{it}w_{it,j} \\ \Rightarrow s_{jt} &:= \left(\frac{ME_{jt}}{ME_t} \right) = \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) w_{it,j}. \end{aligned} \quad (23)$$

This equation tell us that in the aggregate we will compare the observed stock market shares of a “market value portfolio” (s_{jt}) with the model implied shares of a “wealth-adjusted aggregate portfolio” $\left(\sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) w_{it,j} \right)$. To compute the right hand side of (23), the model-implied shares, one challenge is that the investor-specific taste shocks v_{it} are not observed by the econometrician. To deal with this problem it is common to assume a prior distribution on these latent variables. If F_v denotes the distribution of v_{it} we can numerically integrate out its role on portfolio holdings using F_v . The model-implied shares, denoted by \tilde{s}_{jt} , are

$$\tilde{s}_{jt} := \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) \int \frac{\exp(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it}))}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it}))} dF_v(v_{it}). \quad (24)$$

It is standard practice in the IO literature to assume F_v to be a multivariate normal with zero mean and variance-covariance matrix S_v , that is $v_{it} \sim N(0, S_v)$; then the integral in (24) can be numerically approximated for example by monte carlo simulation or using

Gauss-Hermite quadrature procedures. The $\tilde{s}_{jt}(\cdot)$ functions define a demand system

$$\tilde{s}(\delta_t, \theta_2; d_t, X_t, J_t) = (\tilde{s}_1(\delta_t, \theta_2; d_t, X_t, J_t), \dots, \tilde{s}_{J_t}(\delta_t, \theta_2; d_t, X_t, J_t)), \quad (25)$$

The aggregation across investors in (24) with wealth-based weights (A_{it}/A_t) makes this demand system different from the standard RC demand system (for example the canonical BLP demand system of Berry et al. (1995)) but we can prove this system is *invertible* in the sense that given $(\theta_2; d_t, X_t)$ there is an unique vector δ such that for all j :

$$\tilde{s}_{jt}(\delta, \theta_2; d_t, X_t, J_t) = s_{jt},$$

Proposition 4. *Demand Inversion*

The demand system in (25) is invertible such that given $(\theta_2; d_t, X_t, J_t)$ and nonzero market shares s_{jt} with $\sum_{j=1} s_{jt} < 1$, there exists an unique vector δ such that $\tilde{s}_{jt}(\delta, \theta_2; d_t, X_t, J_t) = s_{jt}$, for all j .

The proof is included in Appendix A and it follows by verifying the conditions of the Berry's inversion theorem (Berry (1994)). Such vector δ can be found as the fixed point of the following contraction mapping. Let $f : \mathbb{R}^J \rightarrow \mathbb{R}^J$ that for fixed $(\theta_2; d_t, X_t, J_t)$ is given by

$$f(\delta) = \delta + \log(s_t) - \log(\tilde{s}_t(\delta, \theta_2; d_t, X_t, J_t)), \quad (26)$$

notice that if δ^* is such that $f(\delta^*) = \delta^*$ then $\log(s_t) = \log(\tilde{s}_t(\delta^*, \theta_2; X_t, J_t))$. The fact that the demand system is invertible allow us to operationalize an estimation strategy centered around the term $\tilde{\zeta}_{jt}$ being interpreted as an structural error term.

Using $\tilde{\zeta}_{jt}$ as an structural error term for estimation is predicated on the assumption that unobserved stock characteristics should be conditionally mean zero with respect to a vector z_{jt} of observable stock characteristics,

$$E[\tilde{\zeta}_{jt} | z_{jt}] = 0, \quad (27)$$

in particular this condition implies that unobservable stock characteristics in z_{jt} should not be correlated with unobserved characteristics $\tilde{\zeta}_{jt}$. In the context of asset demand, as KY2019 mention, this moment condition is motivated by the literature of asset pricing in endowment economics (Lucas (1978)) that assumes that shares outstanding and asset characteristics other than price are exogenous.

The fact that the demand system (23) is invertible, allow us to exploit moment (27) for estimation. Given θ_2 we can “invert the demand” to find $\hat{\delta}_t(\theta_2)$ such that model-implied shares $\tilde{s}_t(\hat{\delta}_t, \theta_2)$ match the observed shares, s_t . With $\hat{\delta}_t(\theta_2)$ we can construct $\tilde{\zeta}_t(\theta)$ given by $\tilde{\zeta}_{jt}(\theta) = \hat{\delta}_{jt}(\theta_2) - X'_{jt}\theta_1$ and then we can select $\theta = (\theta_1, \theta_2)$ that minimizes a GMM objective function based on (27). Formally, in the GMM strategy for estimation of the demand system in (25) we look for $\hat{\theta}_{GMM}$ in market t that solves:

$$\min_{\theta} g(\tilde{\zeta}_t(\theta))' W_t g(\tilde{\zeta}_t(\theta)) \quad (28)$$

$$\text{s.t. } g(\tilde{\zeta}_t(\theta)) = \frac{1}{J_t} \sum_{j=1}^{J_t} z_{jt} \tilde{\zeta}(\theta)_{jt} \quad (29)$$

$$\tilde{\zeta}(\theta)_{jt} = \delta_{jt}(\theta_2) - X'_{jt}\theta_1 \quad (30)$$

$$\log(s_{jt}) = \log(\tilde{s}_{jt}(\delta_t, \theta_2; d_t, X_t, J_t)) \quad (31)$$

$$\tilde{s}_{jt}(\delta_t, \theta_2; d_t, X_t, J_t) = \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) \int \frac{\exp(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it}))}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it}))} dF_v(v_{it}) \quad (32)$$

where W_t is a $K_Z \times K_Z$ weight matrix. Using the first order conditions of the GMM problem is possible to show that the parameter search can be simplified to be just over θ_2 (see Berry et al. (1995) and Nevo (2000) for useful derivations). To see why, it suffices to notice that the first order conditions with respect to θ_1 requires

$$\begin{aligned} 0 &= \frac{\partial}{\partial \theta_1} \left(g(\tilde{\zeta}(\theta_t))' W_t g(\tilde{\zeta}(\theta_t)) \right) \quad (33) \\ \Leftrightarrow 0 &= -\frac{2}{J_t^2} [\delta_t(\theta_2) - X_t \beta_{0t}]' Z_t W_t Z_t' X_t \\ \Leftrightarrow \theta_1 &= [X_t' Z_t W_t Z_t' X_t]^{-1} [X_t Z_t W_t Z_t'] \delta_t(\theta_2), \end{aligned}$$

so given a value of θ_2 , there is a corresponding value for θ_1 according to the GMM objective function. If the estimation is carried out using data from multiple market periods but the parameters in θ are common across such periods, the GMM strategy would compute (30) to (32) each period t and then stack the moment analogs (29) for each market before computing the objective function in (28). Algorithm 1 sketches the steps necessary to implement the GMM estimation of the random coefficients demand.

Algorithm 1: Random Coefficients Demand Estimation

Input: Stock characteristics $\{X_{jt} = (x'_{jt}, c'_{jt})'\}$, aggregate stock holdings $\{s_{jt}\}$ and instrumental variables $\{Z_{jt}\}$ for $j = 1, \dots, J_t$; assets under management and demographics $\{A_{it}, d_{it}\}$ for $i = 1, \dots, I_t$ for a given market t .

Output: A set of estimated parameters $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ with $\hat{\theta}_1 = \hat{\beta}_0$ and $\hat{\theta}_2 = (\hat{\Pi}, \hat{\Sigma})$.

Initialize: Pick initial values for $\theta_1^{(0)}$ and $\theta_2^{(0)} = (\Pi^{(0)}, \Sigma^{(0)})$.

During step $r \geq 1$ of the optimization routine, do:

- i. Compute an initial value for $\delta_t^{(r,0)}$. If $r = 1$ the initial value can be $\delta_t^{(r,0)} = X'_{jt}\theta_1^{(0)}$.
- ii. Given the current value for $\theta_2^{(r)}$, invert the demand system using the contraction mapping in (26).

Iterate until convergence an update for δ_t where the h -th update is given by:

$$\delta_t^{(h)}(\theta_2^{(r)}) = \delta_t^{(h-1)} + \log(s_t) - \log\left(\tilde{s}_t\left(\delta_t^{(h-1)}, \theta_2^{(r)}; X_t, J_t\right)\right)$$

Use $\delta_t^{(r,0)}$ for the first update. The resulting vector will be a function of $\theta_2^{(r)}$ and aggregate stock holdings s_t , denoted by $\delta_t^{(r)}$.

- iii. Update the value of the linear parameters using (33)

$$\theta_1^{(r)} = [X'_t Z_t W_t Z'_t X_t]^{-1} [X_t Z_t W_t Z'_t] \delta_t^{(r)}.$$

- iv. Use $\delta_t^{(r)}$ and $\theta_1^{(r)}$ to compute the GMM error term:

$$\tilde{\zeta}_t^{(r)} = \delta_t^{(r)} - X'_t \theta_1^{(r)},$$

and the GMM moment function, notice that g is a function of the parameters in step r :

$$g(\theta^{(r)}) = \frac{1}{J_t} Z'_t \tilde{\zeta}_t^{(r)}.$$

- v. Evaluate the GMM objective function at $\theta^{(r)}$. If the objective function has converged report $\hat{\theta}_{GMM} = \theta^{(r)}$. If no convergence has been achieved update $\theta_2^{(r)}$ according to the optimization algorithm used (e.g. a Newton-Rapson update). Label this update as $\theta_2^{(r+1)}$.
 - vi. Repeat steps i. to v. until convergence of the GMM objective function.
-

Price Endogeneity. Since the unobserved stock characteristics ζ_{jt} are part of investors demand they will also be a determinant of equilibrium prices. This means that prices and functions of prices will be correlated with ζ_{jt} and cannot be included in z_{jt} for estimation. To solve for the endogeneity of prices with respect to ζ_{jt} we need instrumental variables (IVs) correlated with prices but exogenous with respect to ζ_{jt} .

We consider instrumental variables of the style of Gandhi and Houde (2019). For each stock j denote with $J_t(j)$ the set of stocks that belong to j 's industry. Next, for each exogenous dimension k in X_{jt} , we compute a metric of j 's isolation with respect to other stocks in $J_t(j)$:

$$X_{jt,k}^{GH} = \sum_{\tilde{j} \in J_t(j)} (X_{jt,k} - X_{\tilde{j}t,k})^2. \quad (34)$$

Then the vector of instrumental variables z_{jt} used for estimation will be composed of the exogenous characteristics in X_{jt} plus the Gandhi-Houde IVs (GH-IVs) constructed from such exogenous characteristics. If the dimensions considered to be exogenous with respect to ζ_{jt} are in fact exogenous, then the GH-IVs would be uncorrelated with ζ_{jt} by construction since they rely on the values of an exogenous characteristic for j and the corresponding values for those stocks in j 's industry.

The case for relevance is more interesting; since stocks are considered as differentiated investment products, they compete on the characteristics they offer to investors. Those stocks with more attractive characteristics to investors will have a relatively higher demand, all else equal. Then, metrics of j 's isolation with respect to other stocks in $J_t(j)$ along an exogenous characteristic would capture stock j 's ability to compete on such characteristic against alternative stocks in j 's industry. If the alternatives of stock j in its industry offer more (less) of a characteristic positively value by investors relative to j , then there will be more (less) demand for alternatives of stock j , less (more) for stock j itself and that will decrease (increase) the price of stock j . Hence metrics of stock j 's isolation with respect to other stocks in $J_t(j)$ will be correlated with the price of stock j .

Logit demand estimation. When the non-linear parameters θ_2 are restricted to zero, $\theta_2 = (\Pi, \Sigma) = (0, 0)$, we are in the case of logit demand. In this case the demand system

given by \tilde{s}_{jt} in (24) becomes:

$$\begin{aligned}\tilde{s}_{jt} &= \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) \int \frac{\exp(\delta_{jt} + h_{ijt}(0, d_{it}, v_{it}))}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt} + h_{ijt}(0, d_{it}, v_{it}))} dF_v(v_{it}). \\ &= \frac{\exp(\delta_{jt})}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt})} \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t} \right) \\ &= \frac{\exp(\delta_{jt})}{1 + \sum_{j=1}^{J_t} \exp(\delta_{jt})}'\end{aligned}$$

in the second line we used the fact that when $\theta_2 = 0$ then $h_{ijt} \equiv 0$, and in the third line we use the fact that wealth weights should sum up to one. In the case of logit demand we can perform the demand inversion analytically, since $\tilde{s}_{0t} = 1/(1 + \sum_{j=1}^{J_t} \exp(\delta_{jt}))$ then:

$$\log(\tilde{s}_{jt}/\tilde{s}_{0t}) = \delta_{jt} = X'_{jt}\beta_0 + \xi_{jt},$$

the first equation tell us that if we set the value of δ_{jt} to $\log(s_{jt}/s_{0t})$ then observed shares will match the (logit) model-implied shares. The second equation which is the definition of δ_{jt} tell us how to construct ξ_{jt} to use it in a GMM estimation strategy base on moment (27). Specifically, logit estimates will be obtained by linear IV-GMM based on (27) and using the Gandhi-Houde IVs described above.

4 Demand for Green Stocks

In this section we start by presenting the random coefficients demand specification we'll use for estimation. Then we present the data sources and finally we present the demand estimation results.

Following the notation of equation (15), we estimate a demand specification given by

$$w_{it,j} = \frac{\exp(x'_{jt}b_0 + c_{jt}\gamma_{it} + \xi_{jt})}{1 + \sum_{j=1}^{J_t} \exp(x'_{jt}b_0 + c_{jt}\gamma_{it} + \xi_{jt})}, \quad (35)$$

where the vector of return-related characteristics is given by

$$x_{jt} = (1, \text{mktBeta}_{jt}, \text{lat}_{jt}, \text{lbme}_{jt}, \text{profitability}_{jt}, \text{investment}_{jt}),$$

that includes an intercept, market beta (mktBeta_{jt}), log total assets (lat_{jt}), log book-to-market equity (lbme_{jt}), stock's profitability¹⁷ ($\text{profitability}_{jt}$) and stock's investment¹⁸ (investment_{jt}). These return characteristics are motivated by the Fama-French five-factor model (Fama and French (2015)) that offer sensible dimensions to characterize the cross section of returns¹⁹.

The vector of taste characteristics c_{jt} is composed of the environmental scores of company j in t , $c_{jt} = (\text{Escore}_{jt})$. Moreover, the coefficients corresponding to the return-related characteristics are treated as homogeneous across investors.²⁰ The coefficient for environmental scores is modeled as heterogeneous across investors following the structure:

$$\gamma_{it} = \gamma_0 + \kappa d_{it} + \sigma v_{it}, \quad (36)$$

where the parameters $(\gamma_0, \kappa, \sigma)$ are common across investors but each investor has a different sensitivities to the environmental scores of the stocks because they differ in their observed demographics d_{it} and unobserved demographics, or taste shocks, v_{it} . In this demand specification we use the investor's assets under management as observed demographics, $d_{it} = \log(\text{AUM})_{it}$; and we assume unobserved demographics follow a standard normal distribution independent and identically distributed across investors. This distributional assumption is common practice in the demand estimation literature and facilitates the numerical approximation of the integral in the definition of model shares, \tilde{s}_{jt} in (24), during estimation we approximate such integral using a Gauss-Hermite quadrature approximation of order 20²¹.

¹⁷Measured as operating profits to book equity.

¹⁸Measured as annual log growth of total assets.

¹⁹There is a growing literature in the asset pricing questioning whether the Fama-French five-factor characteristics are sufficient to explain the cross section of returns (e.g. Han et al. (2021)). However, considering alternative returns characteristics other than those in the Fama-French five-factor is left for future research.

²⁰Allowing for heterogeneity on return-related characteristics is left for future research.

²¹This guarantees the integral is exact for polynomial functions of degree up to 49.

4.1 Data

There are two main sources of data: stock characteristics and portfolio holdings. Data on portfolio holdings comes from the Thomson Reuters Institutional Holdings Database that contains data on institutional investors that file the Form 13F from the Securities and Exchange Commission (SEC). Investment institutions that manage more than \$100 million are required to disclose stock holdings in the Form 13F. These institutions can be banks, insurance companies, mutual, hedge, and pension funds, as well as other 13F institutions like foundations, nonfinancial corporations, and endowments.

Price and stock characteristics data for this project comes from the Compustat and Center for Research in Security Prices (CRSP) datasets which we combine to obtain fundamentals for publicly traded companies in the US stock market. Data for stock prices, dividends, returns and shares outstanding can be obtained from the CRSP Monthly Stock Database. Accounting data from the Compustat North America Fundamentals Annual and Quaterly Databases are combined with CRSP data to construct asset characteristics.

The data uses common stocks (with share codes 10, 11, 12, 18) that trade in the New York Stock Exchange, the American Stock Exchange and Nasdaq (exchange code 1, 2, 3 respectively); those stocks with missing data on returns or prices are filter out. Data on the CRSP database are merge with Compustat database records most recent of at least 6 months, and no more of 18 months prior to trading date. This is to guarantee that accounting data were public on the trading date.

Data on environmental performance of listed companies comes from the MSCI rating agency²². MSCI is a pioneer rating agency in the construction of scores that evaluate the Environmental, Social, and Governance (ESG) performance of the firms they rate²³. We obtained a dataset of firm-level annual ESG scores from 1991 to 2019. One key advantage of the dataset from MSCI is the availability of granular data, for each of the three pillars:

²²There is a growing number of data providers of ESG scores and a growing literature studying to what extend scores from different vendors are consistent in their evaluations (e.g. Billio et al. (2021)). Using data from other vendors would not alter the methodology here presented, but comparing how results would change when using different ESG score are used is left for future research.

²³Moreover, as mentioned in Pástor et al. (2022), MSCI has been voted “Best firm for SRI research” in the The Extel and SRI Connect Independent Research in Responsible Investment (IRRI) Survey each year from 2015 to 2019 (see <https://www.msci.com/zh/esg-ratings>).

E, S and G, the dataset includes a series of performance indicators used to construct the final score. For most vendors of ESG scores, the final score is the result of translating raw data into a numerical score using a proprietary algorithms; with the MSCI granular data we can construct scores directly from the raw data guided by the application at hand.

We focus on MSCI variables from the “Environmental pillar score”, where firms are evaluated in several indicators that capture either positive or negative environmental performance. Positive indicators include appropriate waste management, product carbon footprint, and energy efficiency, while negative indicators include regulatory compliance, toxic emissions and waste, water stress; see Appendix B for a full list of indicators²⁴. Using these indicator variables on environmental performance we construct a E-score that compares firm cross-sectionally each year according to their environmental performance.

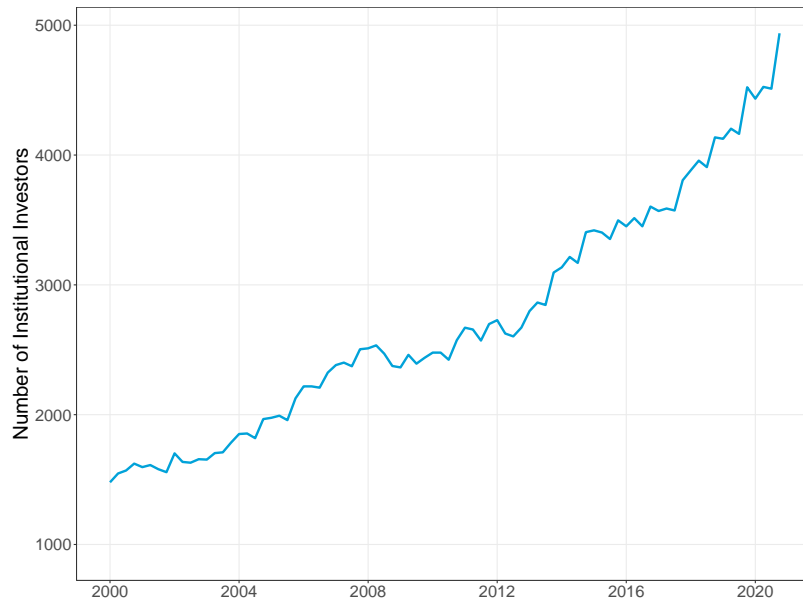
Figure 1 presents the evolution of the number of stocks and institutional investors on the sample period. Panel (b) shows that 13F institutions have grown in importance over the sample period. Towards the end of the sample institutional investor collectively managed around 70% of the US stock market from around 53% at the beginning of the sample. See Appendix B for the distribution of institutional investors according to their type over the sample period.

Moreover in each market, we construct a residual investor labeled as the *household sector*. The stock holdings of the household sector are defined as difference between shares outstanding and the sum of shares held by 13F institutions. The introduction of the household sector is necessary for the market clearing to hold. The outside option, $j = 0$, will be set to include be stocks that are foreign (code 12), real estate investment trusts (code 18) or have missing characteristics or returns.

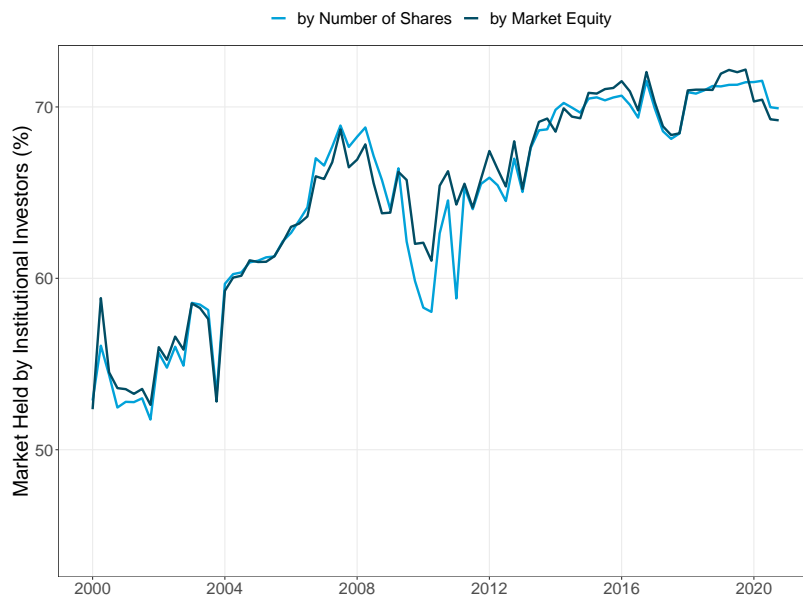
Data Construction. The construction of the return-related characteristics in the vector x_{jt} follows KY2019. It includes five characteristics: market beta, log total assets, log book-to-market equity, a metric of firm’s profitability and a metric of firm’s investment. Market beta for each stock j is compute on a 60-month rolling window where we the

²⁴The threshold for satisfying an indicator are determined by MSCI and are not disclosed with the data.

Figure 1: Institutional Stock Holdings



(a) Number of institutional investors



(b) Market held by institutional investors

Notes: Panel (a) shows the evolution on the number of institutional investors in the dataset from 2000-Q1 to 2020-Q1. Panel (b) shows the how much of the U.S. stock market is held by institutional investors from 2000-Q1 to 2020-Q1. First, by number of shares shows the percentage of shares outstanding that is held by institutional investors across all stocks. Second, by market equity shows the dollar value of stocks held by institutional investor as a percentage of the total market equity across all stocks.

market return comes from Kenneth French's website²⁵ and the risk-free rate from 3-month Treasury bills. Log market equity is computed summing log price per share at the end of the quarter with the log of shares outstanding expressed in millions. To compute the profitability metric we follow Fama and French (2015) and compute operating profits to book equity. Operating profits in turn are computed as total revenue (*revt*) minus the sum of cost of goods sold (*cogs*), selling, general and admin expenses (*xsga*), and interest and related expense-total (*xint*), or $profit = (revt - cogs - xsga - xint)$. The investment metric is the 1-year log growth of total assets. Table 2 in Appendix B shows summary statistics for the return characteristics in x_{jt} grouped by sample's year. To reduce the impact of outliers on some variables, profitability, market beta and investment are windsorized at the 2.5th and 97.5th percentile. This windsorizing is done in a quarterly fashion.

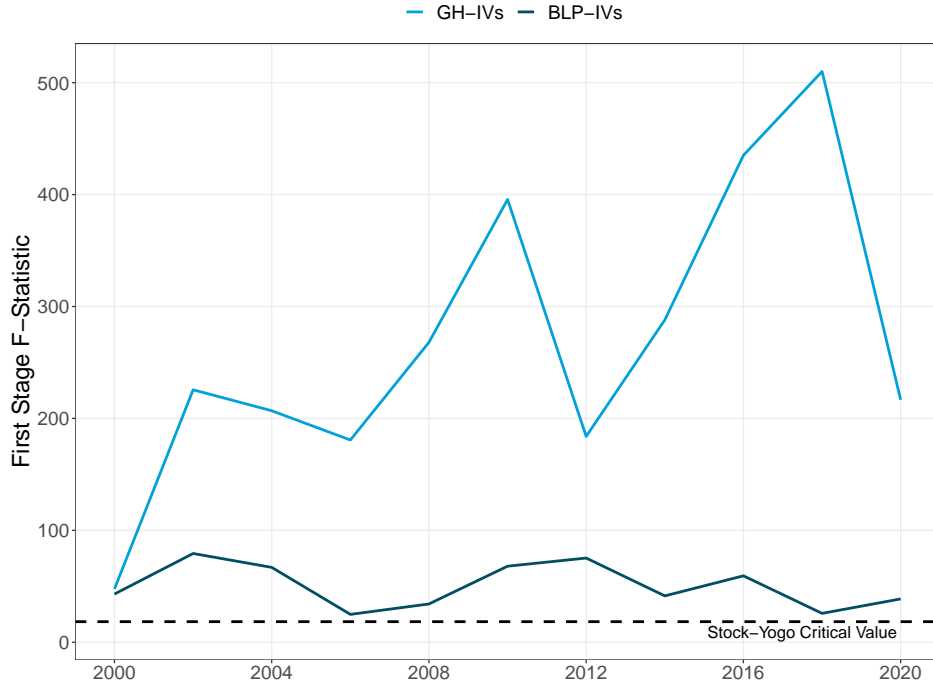
The environmental scores are constructed following the treatment of Engle et al. (2020) and Hong and Kostovetsky (2012). For each firm in the MSCI dataset we count the number of positive indicators and subtract from it the number of negative indicators, we can this difference raw E-scores. Then, after merging the raw score into the dataset of stock holdings and asset characteristics for each quarter, we rank the raw E-scores cross-sectionally and standardize to range in the interval between -1/2 and 1/2; this are the E-scores used for estimation. In this standardization the median raw score is mapped to zero, 1/2 corresponds to the stock with the highest environmental performance, the *greenest* stock, and -1/2 corresponds with the lowest environmental performance, the *brownest* stock. The data on each firm on the MSCI dataset is updated at least once a year, but not all firm scores get update at the same in a given year. To ensure E-scores are public on the trading date, we merge stock holdings and return characteristics in period t with the E-scores from the calendar year prior to t ²⁶.

For estimation, the return-related characteristics other that do not depend on price directly are assumed to exogenous (with respect to ξ_{jt}), that is the market beta, log book equity, profitability and investment. The E-score is also considered as exogenous. On these five characteristics we construct the Gandhi-Houde IVs described above, according to (34). Figure 2 shows the results of a first stage F-test for the null of weak instruments

²⁵http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

²⁶For example, in 2018-Q1 and 2018-Q4 we use E-scores from 2017, whereas in 2019-Q1 we use E-scores from 2018.

Figure 2: First Stage F-stat of the instrumental variables



Notes: First stage F-statistic on the instruments for log Book to Market Equity. We present results for the Gandhi-Houde IVs and the BLP-type IVs. Stock and Yogo (2005) critical value (18.37) for 1 endogenous regressor, 5 instrumental variables and 0.05 bias of two stage least squares relative to OLS. Quarterly sample from 2000-Q1 to 2020-Q4.

across the sample period. The figure also shows the F-statistics when using “BLP”-type instruments, another common choice of instrumental variables in the IO literature²⁷. In all of the estimation windows the F-statistic of the GH-IVs is above the appropriate critical value to reject the null of weak instruments at 5 percent significance level. Moreover, relatively to BLP-IVs, the null of weak IVs is rejected more easily using GH-IVs.

4.2 Estimates

Recall that in estimation the goal is to use data on stock characteristics $\{X_{jt} = (x'_{jt}, c'_{jt})'\}$, aggregate stock holdings $\{s_{jt}\}$, instrumental variables $\{Z_{jt}\}$, and assets under management and demographics $\{A_{it}, d_{it}\}$ to estimate the parameters in θ , composed of linear parameters $\theta_1 = (b_0, \gamma_0)$ and non-linear parameters $\theta_2 = (\kappa, \sigma)$.

²⁷The BLP instruments for the price of stock j are constructed as the sum of the exogenous characteristics of other stocks in j 's industry.

We use twenty years of quarterly data of stocks holdings and characteristics from 2000-Q1 to 2019-Q4. We perform estimation in two-year windows, so for every estimation window we obtain a set of parameter estimates $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ that use data from eight quarters²⁸. Moreover during estimation we standardize the variables log total assets, profitability, investment and E-scores to have mean zero and standard deviation one at each quarterly cross section of stocks. This way the estimated coefficients can be interpreted as the semi-elasticity with respect to the corresponding stock characteristic if multiplied by 100.

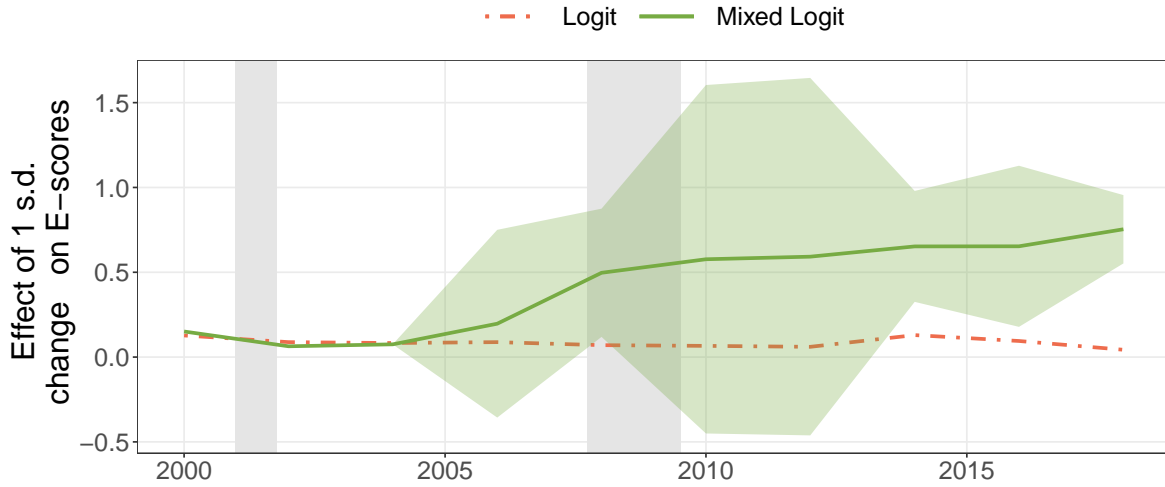
Figure 3 shows the effective coefficient on E-scores, $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\kappa} \log(\text{AUM})_{it} + \hat{\sigma} v_{it}$, over 2-year estimation windows, and according to estimation based on logit demand or random coefficients demand (RC). The plot uses the mean value, in each window, of log assets under management and shows the 95% confidence interval, of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it} , are normally distributed.

From Figure 3 we can see that the sensitivity to E-scores varies over time but it is consistently positive throughout the estimation sample. This is true for both logit and mixed logit estimation. After the Great Recession (2007-Q4 to 2009-Q2) period there is an increase in the range of values for the coefficient on E-scores, due to larger estimated values for $\hat{\sigma}$. This suggests an increase in the heterogeneity in the sensitivity to green characteristics across investors after this period. One possible explanation is that after periods of economic downturn, some investors may be more interested in stocks with higher returns and relatively less interested in the environmental-friendliness of the companies underlying the stocks.

The range of values for the effective coefficient on E-scores after the Great Recession suggests that for some investors the sensitivity is consistent with a preference for brown stocks. For such investors, if there is a determinant of returns not captured by the return-related characteristics of the Fama-French five factor model which is higher for brown stocks, an appetite for returns could explain the preference for brown stocks

²⁸The first estimation window uses data from 2000-Q1 to a 2001-Q4, and the last estimation window uses data from 2018-Q1 to 2019-Q4.

Figure 3: Estimated coefficients for E-scores



Notes: This plot shows the effect on the demand exponent of one standard deviation change on E-scores based on the associated coefficient for E-scores, $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\lambda} \log(\text{AUM})_{it} + \hat{\sigma} v_{it}$. Estimates are obtained over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4, and according to estimation based on logit demand or random coefficients demand (RC). Multiply this effect by 100 approximates the semi-elasticity of portfolio weights with respect to E-scores. The plot uses the mean value, in each window, of log assets under management and shows the 95% confidence interval of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it} , are normally distributed. Recession periods of the US economy are shown as shaded gray regions. The case of logit demand $\hat{\gamma}_{it} = \hat{\gamma}_{0t}^{\text{logit}}$.

in this period.²⁹

From 2005 until the end of the sample, the mixed logit estimates on the coefficient for E-scores is increasing. For example, in the estimation window 2018-2019, the estimated semi-elasticity on the holdings of a stock after a 1 standard deviation increase in its E-score would result in a 73% increase in its holdings, compared with a 48% increase in the 2008-2009 estimation window.

²⁹As presented in Pástor et al. (2021), brown stocks can have positive CAPM alphas and higher expected returns than green stocks because they are more exposed to climate risk. Similarly, Bolton and Kacperczyk (2021) find evidence for a carbon premium, in which companies with higher carbon emissions earned higher returns; they also provide evidence that such carbon premium cannot be explained entirely by traditional risk factors. Moreover, In a related study for mutual funds, Das et al. (2018) found that in the three year period after the Great Recession socially responsible mutual funds exhibited a negative and significant alpha with respect to the Fama-French five factor model. Moreover, they found that funds with lower ESG scores outperformed the fund with high ESG scores during this period.

Figure 3 also shows the coefficient corresponding to E-scores if we perform estimation under a logit demand specification. In the logit case, there is no heterogeneity in the coefficient and all investors share the same sensitivity to the score, $\hat{\gamma}_{it} \equiv \hat{\gamma}_0^{logit}$ for all i . It is clear from the figure that the logit estimates exhibit much less variation and they don't increase in the second part of the sample.

Another takeaway from estimation is that the sensitivity to E-scores depends on the investor's assets under management. An alternative version of Figure 3 is presented in Appendix C. It shows the effective coefficient on E-scores plotted not only at the mean value but also at the 25th and 75th percentile of log assets under management in each estimation window. The main trends and features of the E-scores discussed above do not change by plotting the coefficient under various values of log assets under management; of course in a given estimation window the effective coefficient γ_{it} varies with the log assets under management of investor i according to the coefficient $\hat{\kappa}$. The average across estimation windows in the period 2010-2019 for $\hat{\kappa}$ is positive, so larger investors will be more sensitive to E-scores and will have higher demand for green stocks keeping other coefficients and stock characteristics fixed. This is consistent with [Kojien et al. \(2023\)](#) that find that large investors have a higher demand for stocks with higher environmental scores.³⁰ This is also consistent with the finding in [Pastor et al. \(2023\)](#) that larger investors tend to tilt their portfolio towards green stocks relative to smaller investors.

In estimation we also consider return-related characteristics in the demand for stocks. Figure 4 shows the coefficients corresponding to the return characteristics. For most periods, these characteristics have corresponding coefficients with the same sign. Characteristics like market beta have negative coefficients, which is consistent with the interpretation that market beta captures a basic dimension of risk and that risk is disliked by investors. An estimated negative coefficient for book-to-market equity suggests a preference for growth stocks. In periods of low interest rates, like following the Great Recession, growth stocks may be preferred by investors to value stocks and have larger equity valuations. The sensitivity to profitability and investment peak in periods where we also observe low sensitivity to market beta. This could correspond to a change in in-

³⁰The value of the average across estimation windows is 0.303. The magnitude of this average is not directly comparable with the one reported in [Kojien et al. \(2023\)](#) due to the type of data and demand specification they use.

Table 1: Example Estimated Price Elasticities

Price Change	Portfolio Weight (%)	Elasticities (%)	
		Logit	Mixed Logit
Apple	2.83	-0.5686	-0.5809
Alphabet	1.02	-0.0045	-0.0023
Exxon Mobile	1.02	-0.0045	-0.0013

Notes: Estimated elasticities of the aggregate holdings Apple with respect to the price change of selected stock prices. Data and estimates for 2019Q2 under logit and mixed logit estimation.

investor preferences valuing forward-looking aspects of the firms, such as profitability and investment, relatively more, and valuing backward-looking aspects of the firms, such as market beta, relatively less. Notably the 2008-2009 estimation window that includes the Great Recession period is where we observe the largest confidence intervals around most of the estimates. General uncertainty about stock market performance could be reflected in the relatively large standard errors for that period.

We also report an example of the estimated price elasticities. Table 1 shows the estimated price elasticities of the holdings of Apple according to the market value portfolio with respect to the price change of selected stocks. This example was chosen because in 2019Q2, the market value portfolio has similar weights for Alphabet and Exxon Mobile, approximately 1.02%. As discussed in section 3, under logit demand, the cross price elasticities are proportional to portfolio weights; hence under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks. This is precisely what the estimates for logit show; despite Apple and Alphabet belonging to the same industry while Apple and Exxon Mobile belong different industries. On the other hand, the mixed logit estimates are flexible enough to show a larger degree of complementarity between Apple and Alphabet than between Apple and Exxon Mobile.

Figure 4: Estimated coefficients for return-related characteristics



Notes: This plot shows the estimated coefficients corresponding to the return-related characteristics over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4. Shaded regions represent 95% confidence intervals. Recession periods of the US economy are shown as vertical shaded gray regions.

5 Ban of Green Investing for Pension Funds

In this section we use the estimated demand for green stocks to study the effects of a ban of green investing for pension funds on aggregate holdings and equity prices. This counterfactual policy exercise is motivated by policy initiatives discussed in the US Senate at the beginning of 2023. On March 1st 2023, the US Senate passed a bill to prevent pension fund managers from basing investment decisions on factors like climate change (Morgan (2023)). The bill was eventually vetoed by President Biden 19 days later (Thomas (2023)) but many similar initiatives have been approved in various US States legislatures.

To implement a ban on green investing for pension funds, we first identify the institutional investors that are pension funds and counterfactually make their demand for stocks perfectly inelastic to E-scores. To identify pension funds we use the classification of institutional investors from KY2019. This classification groups institutional investors into 6 categories: banks, insurance companies, mutual funds, pension funds, investment advisors (including hedge funds) and other institutions like foundations, nonfinancial corporations, and endowments. Once the pension funds have been identified, we define the following counterfactual demand curves for a stock j :

$$\tilde{w}_{it,j} = \begin{cases} \frac{\exp(\hat{\delta}_{jt} - \hat{\gamma}_0 c_{jt})}{1 + \sum_{j=1}^I \exp(\hat{\delta}_{jt} - \hat{\gamma}_0 c_{jt})} & \text{if } i \text{ is a pension fund} \\ \frac{\exp(\hat{\delta}_{jt} + h_{ijt}(\hat{\theta}_2, d_i, v_i))}{1 + \sum_{j=1}^I \exp(\hat{\delta}_{jt} + h_{ijt}(\hat{\theta}_2, d_i, v_i))} & \text{otherwise.} \end{cases}$$

In these counterfactual demand curves pension funds are inelastic to E-scores, since in their counterfactual demand the non linear parameters are set to zero and the component corresponding to E-scores in $\hat{\delta}_{jt}$ is offset to zero. In this case pension funds adjust their demand curves as if they no longer derive value from the environmental performance of the stocks. As a consequence of the demand change, prices observed in the data would not clear the market and we need to find counterfactual prices that clear the market. We rely on a market clearing condition (17) to find the new counterfactual prices. Recall that the market clearing conditions states that

$$ME_{jt} = \sum_{i=1}^I A_{it} w_{it,j},$$

and this condition can be expressed as a fixed point in the log vector of prices. Using the counterfactual demand curves, \tilde{w}_{it} , we solve for a vector of log prices p^c such that

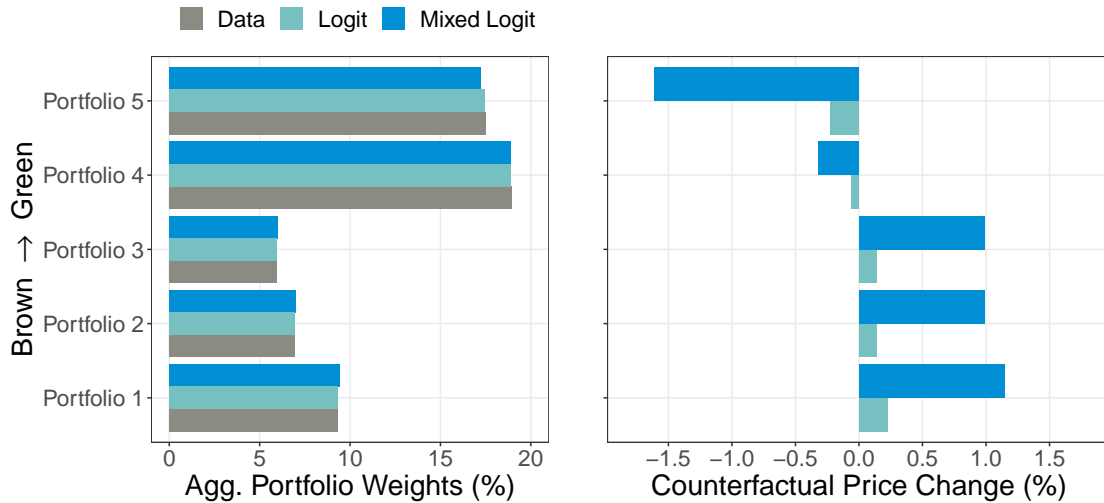
$$p^c = f(p^c) := \log \left(\sum_{i=1}^I A_{it} \tilde{w}_{it}(p^c) \right) - \log(S_t), \quad (37)$$

KY2019 show that a sufficient (but no necessary) condition for this fixed point to exist is that the coefficient accompanying prices has an absolute value less than one. For the period 2019-Q1 where we do the counterfactual exercise the coefficient that corresponding to prices is the coefficient on log book-to-market equity which is -0.443 so it satisfies the sufficient condition for a fixed point to exist.

In computing counterfactual prices that satisfy the market clearing condition the following aspect are assumed to be fixed. First, the number of shares outstanding of each stock is assumed to be fixed, so we have an inelastic supply of the stocks and price changes are determined by demand shifts. Second, the assets under management for each investor A_i is also assumed to remain constant during the counterfactual. That is each investor still decides how to allocate A_{it} dollars into the available stock given that prices change to counterfactual prices and in the case of pension funds they are now inelastic to the E-scores. Third, is it assumed that the coefficients associated to return-related characteristics are fixed as well as the coefficient on E-scores for institutional investors other than pension funds. Fourth, the estimated unobserved stock characteristics, $\hat{\zeta}_{jt}$ is also assumed be remain constant during the counterfactual. It can be argued that for the last two assumptions a [Lucas \(1976\)](#) critique applies to the extent that coefficients on return characteristics and unobserved stock characteristics change with the policy. This critique apply to most counterfactual exercises in the asset demand literature and exploring ways to circumvent the critique is left for future research.

Figure (5) shows the results of the counterfactual exercise using data and estimates for 2019-Q1. The portfolios in the figure were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, the most brown stocks, while Portfolio 5 contains the 20% of stocks with highest E-scores, the most green stocks. The left panel shows aggregate portfolio weights for each portfolio, that is the sum of the market shares for the stocks contained in each portfolio. The aggregate portfolio weights shown are those observed in the data,

Figure 5: Counterfactual holdings and price changes of E-score-based portfolios



Notes: This figure shows the effect of a ban of green investing for pension funds on aggregate holdings and equity prices in a counterfactual exercise using data and estimates for 2019-Q1. The portfolios in the figure were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, while Portfolio 5 contains the 20% of stocks with highest E-scores. The left panel shows the sum of the market shares for the stocks contained in each portfolio. It shows the aggregate portfolio weights observed in the data, under the counterfactual policy using estimates from a random coefficients (RC) demand specification and using estimates of a logit demand specification. The right panel shows the value-weighted average price change for each portfolio comparing the prices observed in the data with the counterfactual prices under RC demand and under logit demand.

under the counterfactual policy using estimates from a random coefficients (RC) demand specification and using estimates of a logit demand specification. Results show that the aggregate holdings in the data are very similar to the counterfactual holdings of logit demand, exhibiting little change. Compared to the counterfactual holdings under the random coefficients demand we see larger differences. Portfolio 5, which is composed of the stocks with the highest E-scores, shows a reduction in aggregate holdings under the policy, whereas Portfolios 1, 2, and 3 increase their aggregate holdings. This means that the relative importance of the stocks in Portfolio 5 diminished while the relative importance of the stocks of portfolios 1, 2 and 3 increased. This suggests that with the policy the aggregate investment share on green stocks was reduced in favor of brown stocks.

The right panel of Figure (5) shows the value-weighted average price change for the

stocks in each portfolio comparing the prices observed in the data with the counterfactual prices under RC demand and under logit demand. Results show that the changes under a logit demand are much smaller than in the random coefficients demand case, this is due to the restrictive elasticities of logit demand where as mentioned before, own-price elasticities are proportional to market shares. The results for RC demand show that Portfolio 5 experienced the most negative change, with an average counterfactual price change of -1.6%, while Portfolio 1 exhibited the biggest positive change with an average counterfactual change of 1.1%³¹. These results have to keep a consistency with the changes in aggregate shares, the price of green stocks will decrease under the new counterfactual prices that clear the market, because there is less demand for green stocks, but the decrease will happen up to a point where the reduction in price no longer encourages more demand of the green stocks and the market clears. From the right panel of Figure (5), as with aggregate shares, the prices of green stocks decrease the most while the price of brown stocks increased with the policy.

The magnitude of the price changes in Figure (5) are commensurate to price changes observed in the data. For example, in the quarter following the data use for the counterfactual, the value-weighted price change of the stocks in Portfolio 1 between 2019-Q1 and 2019-Q2 was 1.1%; a table version of Figure (5) showing the price change over this period for all portfolios is included in Appendix C. Similarly, as studied in Rudebusch et al. (2023), policy announcements that substantially affect green and brown stocks can trigger price changes on stock indices of brown and green stocks ranging from -2% to 11% in a matter of days, as in the case of the Inflation Reduction Act approved in 2022.

6 Conclusions

This paper combines a traditional portfolio construction problem with demand estimation techniques to estimate the demand for green stocks of US institutional investors. In the framework presented, both belief and taste heterogeneity play a role. In addition to investor heterogeneity through differential beliefs about future returns, our framework allows for investors to care about the characteristics of the portfolio they are forming beyond those characteristics related directly to an expected return-versus-risk trade off. We use this framework to measure the preference for green stocks while considering

³¹The corresponding changes for logit demand are -0.2% for Portfolio 5 and 0.2% for Portfolio 1.

return-related stock characteristics.

For estimation this paper uses a mixed logit demand specification in contrast with the logit demand specification more commonly used in the recent asset demand literature. In a logit demand specification, price elasticities are proportional to portfolio shares which restricts the substitution patterns between assets. In a mixed logit demand specification, investor heterogeneity is captured by investor-specific coefficients that are modeled as functions of investor demographics. This richer investor-level heterogeneity delivers more flexible substitution patterns between assets, and it is the modern workhorse model of demand estimation in the IO literature. By doing estimation at the market-level we can not only implement the mixed logit demand specification, but it facilitates dealing with the endogeneity of prices. Specifically, this allows us to consider instrumental variables for prices that have been studied in the IO literature such as BLP-type instruments or Gandhi-Houde price instruments, which have not been used in the asset demand literature.

The empirical exercise uses quarterly data on the stock holdings of institutional investors in the US. We pair this holdings data with return-related characteristics inspired by the Fama-French five factor model and data on the environmental performance of the listed companies in the form of E-scores. We find that the revealed taste for green stocks fluctuates over time. For both logit and mixed logit demand estimation, we find a positive taste for green stocks throughout the estimation sample. However, for mixed logit estimation the semi-elasticity for E-scores increases in the second part of the sample. Moreover, in the period after the Great Recession (2007-Q4 to 2009-Q2) there is an increase in the range of values for the coefficient on E-scores, showing an increase in the heterogeneity in the sensitivity to green characteristics across investors.

In a counterfactual exercise, we use the estimated demand system for stocks to study the effects of a ban on green investing for pension funds on equity prices and aggregate holdings. Using the data and estimates for 2019-Q1, we find that brown stocks will benefit the most in terms of counterfactual pricing. A portfolio with the bottom quintile of green stocks is estimated to have an associated average price change of 1.1% under the counterfactual, while the top quintile portfolio has an average price change of -1.6%.

Future work. Three avenues of work are left for future research. The first one deals with nontraditional stock characteristics. There is a great amount of textual information about listed companies that can be informative to investors. It is possible to extract “topics of risk” by applying a topic model to the text of regulatory risk filings from listed companies (see, e.g. Lopez-Lira (2023)) and using the corresponding topic loadings as a stock characteristic in the demand curves of investors. This would help enrich the demand model with nontraditional but sensible characteristics related to risk. Related work includes Lopez-Lira and Roussanov (2023) that explores whether traditional common factors are enough to explain the cross section of returns.

A second avenue of future research lies at the intersection of asset pricing and corporate finance. As discussed in Brunnermeier et al. (2021) asset demand curves are flexible enough to include firm characteristics such as leverage, innovation, investment, and payout policies as relevant features signaling growth expectations and risks associated with future cash flows. Adding a model of firm corporate policies would complement asset demand systems with models of corporate decision making.

Third, dynamic considerations are at the very frontier of the asset demand estimation literature. Time-conditional statements in asset pricing are paramount and explicitly modeling the time dimension in the demand for stocks would be an important contribution to the literature. It is easy to argue that in practice portfolio optimization in one period is directly related to the stocks held in the previous period, calling for *inventory* considerations when modeling asset demand curves over time. Modeling asset demand dynamically also requires understanding how asset characteristics evolve over time, how investor funds flow in and out of the stock market, and how investor beliefs update over time. All these issues make dynamic asset demand challenging yet exciting for future research.

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Appendices

A Proofs and Mathematical Derivations

A.1 Proof of Proposition 1

Proof. The following proof follows the main steps of Lemma 1 proof's in KY2019 with the adaptation for the more general objective function. The function inside the conditional expectation in (1) takes the form

$$F_i(A_{i,t+1}, C_{it}, w_{it}) = \log(A_{i,t+1}) + a'_i C'_i w_{it}$$

we can replace the first term with

$$\log(A_{i,t+1}) = \log(A_{it}) + \log\left(\frac{A_{i,t+1}}{A_{it}}\right) = \log(A_{it}) + \log\left(R_{t+1}^0 + w'_{it}(R_{t+1} - R_{t+1}^0 \mathbf{1})\right)$$

by using the budget constraint (2). Then the Lagrangian for the problem is given by

$$\begin{aligned} L(w_{it}, \Lambda_{it}, \lambda_{it}) &= \log(A_{it}) + E_{it} \left[\log\left(R_{t+1}^0 + w'_{it}(R_{t+1} - R_{t+1}^0 \mathbf{1})\right) \right] \\ &\quad + a'_i C'_i w_{it} + \Lambda'_{it} w_{it} + \lambda_{it}(1 - \mathbf{1}' w_{it}), \end{aligned}$$

the first order condition with respect to w_{it} given by

$$\begin{aligned} E_{it} \left[\left(R_{t+1}^0 + w_{it}(R_{t+1} - R_{t+1}^0 \mathbf{1}) \right)^{-1} \left(R_{t+1} - R_{t+1}^0 \mathbf{1} \right)' \right] + \Lambda'_{it} - \lambda_{it} \mathbf{1}' + a'_i C'_i &= 0 \\ \Rightarrow E_{it} \left[\left(\frac{A_{i,t+1}}{A_{it}} \right)^{-1} \left(R_{t+1} - R_{t+1}^0 \mathbf{1} \right) \right] &= -(\Lambda_{it} - \lambda_{it} \mathbf{1} + C_t a_i). \quad (\text{A1}) \end{aligned}$$

Multiplying this equation by $-1w'_{it}$ yields:

$$\begin{aligned} -E_{it} \left[\left(\frac{A_{i,t+1}}{A_{it}} \right)^{-1} \mathbf{1} w'_{it} \left(R_{t+1} - R_{t+1}^0 \mathbf{1} \right) \right] &= \mathbf{1} w'_{it} (\Lambda_{it} - \lambda_{it} \mathbf{1} + C_t a_i) \\ \Rightarrow -E_{it} \left[\left(\frac{A_{i,t+1}}{A_{it}} \right)^{-1} \left(\frac{A_{i,t+1}}{A_{it}} - R_{t+1}^0 \right) \mathbf{1} \right] &= \mathbf{1} w'_{it} (\Lambda_{it} - \lambda_{it} \mathbf{1} + C_t a_i) \end{aligned}$$

using the intertemporal budget constraint. Summing the last expression with (A1) results in Euler equation in Proposition 1:

$$E_{it} \left[\left(\frac{A_{it+1}}{A_{it}} \right)^{-1} R_{t+1} \right] = 1 - (I - 1w'_{it}) (\Lambda_{it} - \lambda_{it}1 + C_t a_i).$$

Next, using the intertemporal budget constraint we can write the objective function of the investor problem as:

$$\log(A_{it}) + E_{it} \left[\log \left(R_{t+1}^0 + w'_{it}(R_{t+1} - R_{t+1}^0 1) \right) \right] + a'_i C'_t w_{it}.$$

Let R_{t+1}^p denote the gross return of the portfolio with weights w_{it} , then

$$R_{t+1}^p = R_{t+1}^0 + w'_{it}(R_{t+1} - R_{t+1}^0 1),$$

and the log excess return of the portfolio with respect to the outside option gross return is given by

$$r_{t+1}^p - r_{t+1}^0 = \log \left(\frac{R_{t+1}^p}{R_{t+1}^0} \right) = \log \left(1 + w'_{it} \left(\exp(r_{t+1} - r_{t+1}^0 1) - 1 \right) \right).$$

Now consider the function $g(x) : \mathbb{R}^J \rightarrow \mathbb{R}$ given by $f(x) = \log(1 + w'(\exp(x) - 1))$, where the $\exp(\cdot)$ applies entry-by-entry, and w is a J -vector constant, then a second order Taylor approximation of g around $x_0 = 0 \in \mathbb{R}^J$ is given by

$$g(x) \approx w' \left(x + \frac{1}{2} x \odot x \right) - \frac{1}{2} w' (x x') w,$$

where \odot stands for entry-by-entry multiplication. Applying this approximation to the expression for $r_{t+1}^p - r_{t+1}^0$ yields

$$\begin{aligned} r_{t+1}^p - r_{t+1}^0 &\approx w'_{it} \left((r_{t+1} - r_{t+1}^0) + \frac{1}{2} (r_{t+1} - r_{t+1}^0) \odot (r_{t+1} - r_{t+1}^0) \right) \\ &\quad - \frac{1}{2} w'_{it} (r_{t+1} - r_{t+1}^0) (r_{t+1} - r_{t+1}^0)' w_{it}. \end{aligned}$$

Next we apply the expectations operator $E_{it}[\cdot]$ and the second term in the objective

function can be approximated by

$$\begin{aligned} E_{it} \left[\log \left(R_{t+1}^0 - w'_{it} (R_{t+1} - R_{t+1}^0 \mathbf{1}) \right) \right] &\approx r_{t+1}^0 + w'_{it} \left(E_{it} [r_{t+1} - r_{t+1}^0] + \frac{\tilde{\sigma}_{it}^2}{2} \right) - \frac{w'_{it} \tilde{\Sigma}_{it} w_{it}}{2} \\ &= r_{t+1}^0 + w'_{it} \tilde{\mu}_{it} - \frac{w'_{it} \tilde{\Sigma}_{it} w_{it}}{2}, \end{aligned}$$

this approximation replaces $E_{it}[(r_{t+1} - r_{t+1}^0)(r_{t+1} - r_{t+1}^0)']$ with $\tilde{\Sigma}_{it}$ and $E_{it}[(r_{t+1} - r_{t+1}^0) \odot (r_{t+1} - r_{t+1}^0)]$ with $\tilde{\sigma}_{it}^2$ and follows from the one presented in [Campbell and Viceira \(2002\)](#) (Eq. 2.23). With this approximation the first order condition for becomes

$$\begin{aligned} \tilde{\mu}_{it} - \tilde{\Sigma}_{it} w_{it} + \Lambda_{it} - \lambda_{it} \mathbf{1} + C_t a_i &= 0 \\ \Rightarrow w_{it} &= \tilde{\Sigma}_{it}^{-1} (\tilde{\mu}_{it} + \Lambda_{it} - \lambda_{it} \mathbf{1} + C_t a_i), \end{aligned}$$

Partition the asset space between those with non-binding short sale constraint and those binding we write $\Lambda'_{it} = [0' \quad \Lambda_{it}^{(2)'}]$ and $(C_t a_i) = [(C_t a_i)'_1 \quad (C_t a_i)'_2]'$, then using the partitions for $\tilde{\Sigma}_{it}$ and $\tilde{\mu}_{it}$ we have that

$$w_{it} = \begin{pmatrix} w_{it}^{(1)} \\ 0 \end{pmatrix} = \begin{pmatrix} \Sigma_{it} & \Sigma_{it}^{(1,2)} \\ \Sigma_{it}^{(2,1)} & \Sigma_{it}^{(2,2)} \end{pmatrix}^{-1} \left(\begin{pmatrix} \mu_{it} \\ \mu_{it}^{(2)} \end{pmatrix} + \begin{pmatrix} 0 \\ \Lambda_{it}^{(2)} \end{pmatrix} - \lambda_{it} \mathbf{1} + \begin{pmatrix} (C_t a_i)_1 \\ (C_t a_i)_2 \end{pmatrix} \right).$$

The inverse of $\tilde{\Sigma}_{it}$ is given by

$$\tilde{\Sigma}_{it}^{-1} = \begin{pmatrix} \left(\Sigma_{it} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right)^{-1} & -\Sigma_{it}^{-1} \Sigma_{it}^{(1,2)} \left(\Sigma_{it}^{(2,2)} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{-1} \Sigma_{it}^{(2,1)} \right)^{-1} \\ -\Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \left(\Sigma_{it} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right)^{-1} & \left(\Sigma_{it}^{(2,2)} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{-1} \Sigma_{it}^{(2,1)} \right)^{-1} \end{pmatrix},$$

then w_{it} becomes

$$\begin{pmatrix} w_{it}^{(1)} \\ 0 \end{pmatrix} = \begin{pmatrix} \left(\Sigma_{it} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right)^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1) - \Sigma_{it}^{-1} \Sigma_{it}^{(1,2)} \left(\Sigma_{it}^{(2,2)} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{-1} \Sigma_{it}^{(2,1)} \right)^{-1} (\mu_{it}^{(2)} + \Lambda_{it}^{(2)} - \lambda_{it} \mathbf{1} + (C_t a_i)_2) \\ -\Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \left(\Sigma_{it} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right)^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1) + \left(\Sigma_{it}^{(2,2)} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{-1} \Sigma_{it}^{(2,1)} \right)^{-1} (\mu_{it}^{(2)} + \Lambda_{it}^{(2)} - \lambda_{it} \mathbf{1} + (C_t a_i)_2) \end{pmatrix}.$$

We can multiply the second block by $\Sigma_{it}^{-1} \Sigma_{it}^{(1,2)}$ and sum both blocks to obtain

$$\begin{aligned} w_{it}^{(1)} &= \left(I - \Sigma_{it}^{-1} \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right) \left(\Sigma_{it} - \Sigma_{it}^{(1,2)} \Sigma_{it}^{(2,2)-1} \Sigma_{it}^{(2,1)} \right)^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1) \\ &= \Sigma_{it}^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1). \end{aligned}$$

So following the notation, for the optimal positive weights on the investor's problem can be approximated by

$$w_{it} \approx \Sigma_{it}^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1).$$

To pin down the value of λ_{it} , notice that when constraint (3) is binding then

$$\mathbf{1}' w_{it} = \mathbf{1}' \Sigma_{it}^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_i) = 1,$$

then

$$\lambda_{it} = \frac{\max\{\mathbf{1}' \Sigma_{it}^{-1} (\mu_{it} + C_t a_i) \mathbf{1}, 0\}}{\mathbf{1}' \Sigma_{it}^{-1} \mathbf{1}}.$$

□

A.2 Derivation of investor-specific posterior moments

Proof. We have that

$$\begin{aligned} s_{it} | (r_{t+1}^x, \alpha_i) &\sim N(\alpha_i r_{t+1}^x, \Sigma_\varepsilon) \\ r_{t+1}^x &\sim N(\mu_{xt}, \Sigma_{xt}) \end{aligned}$$

where in our case $\Sigma_\varepsilon = \sigma_\varepsilon^2 I$. The pdfs of these distributions are given by:

$$\begin{aligned} p(s_{it} | r_{t+1}^x, \alpha_i) &= (2\pi)^{-J_t/2} \det(\Sigma_\varepsilon)^{-1/2} \exp \left[-\frac{1}{2} (s_{it} - \alpha_i r_{t+1}^x)' \Sigma_\varepsilon^{-1} (s_{it} - \alpha_i r_{t+1}^x) \right] \\ p(r_{t+1}^x) &= (2\pi)^{-J_t/2} \det(\Sigma_{xt})^{-1/2} \exp \left[-\frac{1}{2} (r_{t+1}^x - \mu_{xt})' \Sigma_{xt}^{-1} (r_{t+1}^x - \mu_{xt}) \right]. \end{aligned}$$

By Bayes theorem $p(r_{t+1}^x | s_{it}, \alpha_i) \propto p(s_{it} | r_{t+1}^x, \alpha_i) p(r_{t+1}^x)$ and

$$\begin{aligned}
\log(p(r_{t+1}^x | s_{it}, \alpha_i)) &= -\frac{1}{2}(s_{it} - \alpha_i r_{t+1}^x)' \Sigma_\varepsilon^{-1} (s_{it} - \alpha_i r_{t+1}^x) \\
&\quad - \frac{1}{2}(r_{t+1}^x - \mu_{xt})' \Sigma_{xt}^{-1} (r_{t+1}^x - \mu_{xt}) + \text{cons} \\
&= -\frac{1}{2}(\alpha_i r_{t+1}^x)' \Sigma_\varepsilon^{-1} (\alpha_i r_{t+1}^x) + (\alpha_i r_{t+1}^x)' \Sigma_\varepsilon^{-1} (s_{it}) \\
&\quad - \frac{1}{2}(r_{t+1}^x)' \Sigma_{xt}^{-1} (r_{t+1}^x) - \frac{1}{2}(r_{t+1}^x)' \Sigma_{xt}^{-1} (\mu_{xt}) + \text{cons} \\
&= -\frac{1}{2}(r_{t+1}^x)' [\alpha_i^2 \Sigma_\varepsilon^{-1} + \Sigma_{xt}^{-1}] (r_{t+1}^x) + (r_{t+1}^x)' [\Sigma_\varepsilon^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt}] + \text{cons} \\
&= -\frac{1}{2} \left(r_{t+1}^x - [\alpha_i^2 \Sigma_\varepsilon^{-1} + \Sigma_{xt}^{-1}]^{-1} [\Sigma_\varepsilon^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt}] \right)' \\
&\quad \cdot [\alpha_i^2 \Sigma_\varepsilon^{-1} + \Sigma_{xt}^{-1}]^{-1} \left(r_{t+1}^x - [\alpha_i^2 \Sigma_\varepsilon^{-1} + \Sigma_{xt}^{-1}]^{-1} [\Sigma_\varepsilon^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt}] \right) + \text{cons}.
\end{aligned}$$

This is the pdf of a multivariate normal distribution with variance $\Sigma_{r|s_i}$ and mean $\mu_{r|s_i}$ given by

$$\begin{aligned}
\Sigma_{r|s_i} &= [\alpha_i^2 (\sigma_\varepsilon^2)^{-1} I + \Sigma_{xt}^{-1}]^{-1} \\
\mu_{r|s_i} &= \Sigma_{r|s_i} [(\sigma_\varepsilon^2)^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt}],
\end{aligned}$$

where we used the fact that $\Sigma_\varepsilon = \sigma_\varepsilon^2 I$. □

A.3 Proof of Proposition 2

Proof. Recall that $\Sigma_{it} = \Sigma_{r|s_i} = (\alpha_i^2 (\sigma_\varepsilon^2)^{-1} I + \Sigma_{xt}^{-1})^{-1}$. The term Σ_{xt} is given by

$$\Sigma_x = (\Gamma_{xt} \Gamma'_{xt} + \sigma_e^2 I)$$

then by the Woodbury matrix identity

$$\begin{aligned}
\Sigma_{xt}^{-1} &= [\Gamma_{xt} \Gamma'_{xt} + \sigma_e^2 I]^{-1} \\
&= \frac{1}{\sigma_e^2} \left(I - \frac{\Gamma_{xt} \Gamma'_{xt}}{\sigma_e^2 + \Gamma'_{xt} \Gamma_{xt}} \right),
\end{aligned}$$

and by substituting into the expression for Σ_{it} yields

$$\begin{aligned}\Sigma_{it}^{-1} &= \left(\frac{\alpha_i}{\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} \right) I - \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})} \Gamma_{xt}\Gamma'_{xt} \\ &= \left(\frac{\alpha_i\sigma_e^2 + \sigma_\varepsilon^2}{\sigma_\varepsilon^2\sigma_e^2} \right) I - \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})} \Gamma_{xt}\Gamma'_{xt} \\ &= \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})} \left[\frac{(\alpha_i\sigma_e^2 + \sigma_\varepsilon^2)(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})}{\sigma_\varepsilon^2} I - \Gamma_{xt}\Gamma'_{xt} \right]\end{aligned}$$

now let's define $\delta_i = \frac{(\alpha_i\sigma_e^2 + \sigma_\varepsilon^2)(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})}{\sigma_\varepsilon^2}$ then

$$\Sigma_{it}^{-1} = \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})} [\delta_i I - \Gamma_{xt}\Gamma'_{xt}]$$

and

$$\begin{aligned}\Sigma_{it} &= \sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt}) [\delta_i I - \Gamma_{xt}\Gamma'_{xt}]^{-1} \\ &= \sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt}) \frac{1}{\delta_i} \left[I + \frac{\Gamma_{xt}\Gamma'_{xt}}{\delta_i - \Gamma'_{xt}\Gamma_{xt}} \right] \\ &= \frac{\sigma_\varepsilon^2\sigma_e^2}{\alpha_i\sigma_e^2 + \sigma_\varepsilon^2} I + \frac{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})}{\delta_i - \Gamma'_{xt}\Gamma_{xt}} \Gamma_{xt}\Gamma'_{xt} \\ &:= l_{it}I + \Gamma_{it}\Gamma'_{it}\end{aligned}$$

with

$$\begin{aligned}l_{it} &:= \frac{\sigma_\varepsilon^2\sigma_e^2}{\alpha_i\sigma_e^2 + \sigma_\varepsilon^2} \\ \Gamma_{it} &:= \left[\frac{\sigma_e^2(\sigma_e^2 + \Gamma'_{xt}\Gamma_{xt})}{\delta_i - \Gamma'_{xt}\Gamma_{xt}} \right]^{1/2} \Gamma_{xt} = \left[\frac{\sigma_\varepsilon^2\sigma_e^2}{\alpha_i\sigma_e^2 + \sigma_\varepsilon^2} \right]^{1/2} \Gamma_{xt} = l_{it}^{1/2}\Gamma_{xt}\end{aligned}$$

□

A.4 Proof of Proposition 3

Proof. Part (i)

First lets work with Γ_{it} , we have that for its j -th entry

$$\Gamma_{it,j} = l_{it}^{1/2} \Gamma_{xt,j} = y'_{jt} \Phi_{\Gamma,i} + \phi_{\Gamma,i}$$

with

$$\begin{aligned}\Phi_{\Gamma,i} &= l_{it}^{1/2} \Phi_{\Gamma} \\ \phi_{\Gamma,i} &= l_{it}^{1/2} \phi_{\Gamma}\end{aligned}$$

Then we work with $\mu_{it,j}$. Recall that $\mu_{it} = \mu_{r|s_i} + \frac{1}{2} \text{diag}(\Sigma_{it})$ and that the term $\mu_{r|s_i}$ is given by

$$\mu_{r|s_i} = \Sigma_{r|s_i} \left[(\sigma_{\varepsilon}^2)^{-1} \alpha_i s_{it} + \Sigma_{xt}^{-1} \mu_{xt} \right]$$

first we need to write $\mu_{r|s_i}$ as a function of μ_{xt}, Γ_{xt} and Γ_{it} . Notice that the term $\Sigma_{xt}^{-1} \mu_{xt}$ can be written as

$$\begin{aligned}\Sigma_{xt}^{-1} \mu_{xt} &= [\sigma_e^2 I + \Gamma_{xt} \Gamma'_{xt}]^{-1} \mu_{xt} \\ &= \frac{1}{\sigma_e^2} \left[I - \frac{\Gamma_{xt} \Gamma'_{xt}}{\sigma_e^2 + \Gamma'_{xt} \Gamma_{xt}} \right] \mu_{xt}\end{aligned}$$

Next we can approximate s_{it} with $\alpha_i \mu_{xt}$ in order to write it as a component that depends on i and μ_{xt} . Here we are using that $E[s_{it}] = \alpha_i \mu_{xt}$. Then we can write

$$\begin{aligned}\mu_{r|s_i} &= \Sigma_{r|s_i} \left[(\sigma_{\varepsilon}^2)^{-1} \alpha_i s_{it} + \Sigma_x^{-1} \mu_x \right] \\ &= [l_{it} I + \Gamma_{it} \Gamma'_{it}] \left[\left(\frac{\alpha_i^2}{\sigma_e^2} + \frac{1}{\sigma_e^2} \right) \mu_{xt} - \left(\frac{\Gamma'_{xt} \mu_{xt}}{\sigma_e^2 + \Gamma'_{xt} \Gamma_{xt}} \right) \Gamma_{xt} \right] \\ &= \kappa_{1i} \mu_{xt} + \kappa_{2i} \Gamma_{xt} + \kappa_{3i} \Gamma_{it}\end{aligned}$$

with

$$\begin{aligned}\kappa_{1i} &= l_{it} \left(\frac{\alpha_i^2}{\sigma_e^2} + \frac{1}{\sigma_e^2} \right) \\ \kappa_{2i} &= -l_{it} \left(\frac{\Gamma'_{xt} \mu_{xt}}{\sigma_e^2 + \Gamma'_{xt} \Gamma_{xt}} \right) \\ \kappa_{3i} &= \Gamma'_{it} \left[\left(\frac{\alpha_i^2}{\sigma_e^2} + \frac{1}{\sigma_e^2} \right) \mu_{xt} - \left(\frac{\Gamma'_{xt} \mu_{xt}}{\sigma_e^2 + \Gamma'_{xt} \Gamma_{xt}} \right) \Gamma_{xt} \right]\end{aligned}$$

then we have that for asset j

$$\begin{aligned}
\mu_{r|s_i,j} &= \kappa_{1i}\mu_{xt,j} + \kappa_{2i}\Gamma_{xt,j} + \kappa_{3i}\Gamma_{it,j} \\
&= \kappa_{1i}(y'_{jt}\Phi_\mu + \phi_\mu) + \kappa_{2i}(y'_{jt}\Phi_\Gamma + \phi_\Gamma) + \kappa_{3i}(y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i}) \\
&= y'_{jt} [\kappa_{1i}\Phi_\mu + \kappa_{2i}\Phi_\Gamma + \kappa_{3i}\Phi_{\Gamma,i}] + [\kappa_{1i}\phi_\mu + \kappa_{2i}\phi_\Gamma + \kappa_{3i}\phi_{\Gamma,i}].
\end{aligned}$$

Recall that $\sigma_{it}^2 = \text{diag}(\Sigma_{it})$, so $\sigma_{it,j}^2 = \Gamma_{it,j}^2 + \iota_{it}$. Since $\Gamma_{it,j}$ is a polynomial of degree M is clear that $\sigma_{it,j}^2$ is a polynomial of degree $2M$ on x_{jt} . To accommodate this we can define \bar{y}_{jt} so it includes the degree combinations of $y_{jt} \otimes y_{jt}$, that is define

$$\bar{y}_{jt} = \begin{pmatrix} x_{jt} \\ x_{jt} \otimes x_{jt} \\ x_{jt} \otimes x_{jt} \otimes x_{jt} \\ \vdots \end{pmatrix}$$

with \bar{y}_{jt} having dimension $K_{\bar{y}} = \sum_{m=1}^{2M} K_x^m$. Then we define $\Phi_{\bar{y}}$ so $(y'_{jt}\Phi_{\Gamma,i})^2 = \bar{y}'_{jt}\Phi_{\bar{y}}$. With this notation we can write

$$\begin{aligned}
\sigma_{it,j}^2 &= \Gamma_{it,j}^2 + \iota_{it} = (y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i})^2 + \iota_{it} \\
&= ((y'_{jt}\Phi_{\Gamma,i})^2) + y'_{jt}[2\phi_{\Gamma,i}\Phi_{\Gamma,i}] + [\phi_{\Gamma,i}^2 + \iota_{it}] \\
&= \bar{y}'_{jt}\Phi_{\bar{y}} + y'_{jt}[2\phi_{\Gamma,i}\Phi_{\Gamma,i}] + [\phi_{\Gamma,i}^2 + \iota_{it}].
\end{aligned}$$

Notice that the term $\mu_{r|s_i,j}$ can be written as a polynomial on \bar{y}_{jt} ; just pad with zeros the coefficients corresponding to powers of the elements in x_{jt} present \bar{y}_{jt} but not in y_{jt} . Then it is possible to write

$$\begin{aligned}
\mu_{it,j} &= \mu_{r|s_i,j} + \frac{1}{2}\sigma_{it,j}^2 \\
&:= \bar{y}_{jt}\Phi_{\mu,i} + \phi_{\mu,i}
\end{aligned}$$

with $\Phi_{\mu,i}$ exact configuration of padded zeros depends on the dimensions of x_{jt} and the degree M and $\phi_{\mu,i}$ is given by

$$\phi_{\mu,i} = \kappa_{1i}\phi_\mu + \kappa_{2i}\phi_\Gamma + \left(\kappa_{3i} + \frac{1}{2}\right)\phi_{\Gamma,i} + \frac{1}{2}\iota_{it}.$$

Part (ii)

Recall that the positive optimal portfolio weights are given by

$$w_{it} \approx \Sigma_{it}^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_{it})$$

using the expression for Σ_{it} we have

$$\begin{aligned} w_{it} &\approx [\iota_{it} + \Gamma_{it} \Gamma'_{it}]^{-1} (\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_{it}) \\ &= \frac{1}{\iota_{it}} \left[I - \frac{\Gamma_{it} \Gamma'_{it}}{\iota_{it} + \Gamma'_{it} \Gamma_{it}} \right] (\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_{it}) \\ &= \left(\frac{1}{\iota_{it}} \right) \mu_{it} - \left(\frac{\lambda_{it}}{\gamma_{it}} \right) \mathbf{1} + C_t \left(\frac{a_{it}}{\iota_{it}} \right) + \kappa_{it} \Gamma_{it} \end{aligned}$$

with

$$\kappa_{it} = - \frac{\Gamma'_{it} (\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_{it})}{\iota_{it} + \Gamma'_{it} \Gamma_{it}}$$

Now define $\tilde{x}_{jt} = [x'_{jt} \quad c'_{jt}]'$ a $K_{\tilde{x}} = (K_x + K_c)$ vector and define the $K_{\tilde{y}}$ vector \tilde{y}_{jt} with $K_{\tilde{y}} = \sum_{m=1}^{2M} (K_x + K_c)^m$ and

$$\tilde{y}_{jt} = \begin{pmatrix} \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \vdots \end{pmatrix},$$

next we show that each term in $w_{it,j}$ can be written as polynomial in \tilde{y}_{jt} . First

$$c'_{jt} \left(\frac{a_{it}}{\iota_{it}} \right) = \tilde{y}'_{jt} \begin{pmatrix} 0 \\ a_{it}/\iota_{it} \\ 0 \\ \vdots \end{pmatrix} = \tilde{y}'_{jt} \Phi_{C,i}$$

next

$$\begin{aligned}
\left(\frac{1}{l_{it}}\right) \mu_{it,j} &= \left(\frac{1}{l_{it}}\right) (\tilde{y}'_{jt} \Phi_{\mu,i} + \phi_{\mu,i}) \\
&= \tilde{y}'_{jt} \left(\frac{1}{l_{it}} \Phi_{\mu,i}\right) + \frac{\phi_{\mu,i}}{l_{it}} \\
&= \tilde{y}'_{jt} \tilde{\Phi}_{\mu,i} + \frac{\phi_{\mu,i}}{l_{it}}
\end{aligned}$$

where $\tilde{\Phi}_{\mu,i}$ has zeros whenever a term with c_{jt} appears in \tilde{y}_{jt} . Finally,

$$\begin{aligned}
\kappa_{it} \Gamma_{it,j} &= \kappa_{it} (y'_{jt} \Phi_{\Gamma,i} + \phi_{\Gamma,i}) \\
&= y'_{jt} (\kappa_{it} \Phi_{\Gamma,i}) + \kappa_{it} \phi_{\Gamma,i} \\
&= \tilde{y}'_{jt} \tilde{\Phi}_{\Gamma,i} + \kappa_{it} \phi_{\Gamma,i},
\end{aligned}$$

and once again $\tilde{\Phi}_{\Gamma,i}$ has zeros whenever there is term in \tilde{y}_{jt} with c_{jt} or has a power of x_{jt} not present in y_{jt} . Collecting terms we have that

$$w_{it,j} \approx \tilde{y}'_{jt} \Phi_{w,it} + \phi_{w,it}$$

with

$$\begin{aligned}
\Phi_{w,it} &= \tilde{\Phi}_{\mu,i} + \Phi_{C,i} + \tilde{\Phi}_{\Gamma,i} \\
\phi_{w,it} &= \frac{\phi_{\mu,i}}{l_{it}} + \kappa_{it} \phi_{\Gamma,i} - \lambda_{it}.
\end{aligned}$$

Part (iii)

Restricting the parameters so $\phi_{w,it} = w_{it0}$ and $\Phi_{w,it}/w_{it0} = [\beta_{it} \quad 1/2 \text{vec}(\beta_{it} \beta'_{it}) \quad \dots]'$ then

$$\begin{aligned}
\frac{w_{ij,t}}{w_{it,0}} &\approx 1 + \tilde{y}'_{jt} \frac{\Phi_{w,it}}{w_{it0}} = 1 + \tilde{x}'_{jt} \beta_{it} + \frac{1}{2} \text{vect}(\tilde{x}_{jt} \tilde{x}'_{jt})' \text{vec}(\beta_{it} \beta'_{it}) + \dots \\
&= \sum_{m=1}^M \frac{(\tilde{x}'_{jt} \beta_{it})^m}{m!} \rightarrow \exp[\tilde{x}'_{jt} \beta_{it}], \quad \text{as } m \rightarrow \infty.
\end{aligned}$$

Writing $\beta'_{it} = [1 \quad b'_{it} \quad \gamma'_{it}]$ and assuming the first characteristics in x_{jt} is unobserved and

denote by ζ_{jt} then we have that

$$\frac{w_{it,j}}{w_{it,0}} \approx \exp\left(\zeta_{jt} + x'_{jt}b_{it} + c'_{jt}\gamma_{it}\right)$$

Finally because $w_{it,0} + \sum_{j=1}^{J_t} w_{it,j} = 1$, then $1 + \sum_{j=1}^{J_t} w_{it,j}/w_{it,0} = 1/w_{it,0}$ and

$$w_{it,j} \approx \frac{\exp\left(x'_{jt}b_{it} + c'_{jt}\gamma_{it} + \zeta_{jt}\right)}{1 + \sum_{k=1}^{J_t} \exp\left(x'_{kt}b_{it} + c'_{kt}\gamma_{it} + \zeta_{kt}\right)},$$

and the weight for the outside option is given by

$$w_{it,0} \approx \frac{1}{1 + \sum_{k=1}^{J_t} \exp\left(x'_{kt}b_{it} + c'_{kt}\gamma_{it} + \zeta_{kt}\right)}.$$

□

A.5 Proof of Proposition 4

We start by stating Berry's Inversion theorem for demand systems. See [Berry \(1994\)](#) for a full proof. Then the proof consists in verifying the conditions of the theorem for the demand system in (25).

Berry's Inversion Theorem Consider the metric space (\mathbb{R}^K, d) with $d(x, y) = \|x - y\|$ and $\|\cdot\|$ denoting the sup-norm. Let $f : \mathbb{R}^K \rightarrow \mathbb{R}^K$ satisfy:

- i. $\forall x \in \mathbb{R}^K$, $f(x)$ is continuously differentiable such that for any j and k :

$$\begin{aligned} \frac{\partial f_j(x)}{\partial x_k} &\geq 0 \\ \sum_{k=1}^K \frac{\partial f_j(x)}{\partial x_k} &< 1 \end{aligned}$$

- ii. $\min_j \inf_x f(x) := \underline{x} > -\infty$

- iii. There is a value \bar{x} with the property that if for any j $x_j \geq \bar{x}$ then for some k (not necessarily equal j) $f_k(x) < x_k$.

Then there is a unique fixed point $x_0 \in \mathbb{R}^K$ to f . Moreover, let $\mathcal{X} := [\underline{x}, \bar{x}]^K$ and define the truncated function $\hat{f}_j(x) = \min\{f_j(x), \bar{x}\}$. Then $\hat{f}(x)$ is a contraction of modulus less than one on \mathcal{X} .

Proof. Varying the conditions of Berry's Inversion Theorem

Denote by θ the parameters to estimate in market t ; s_t the vector of observed aggregate share, and $\tilde{s}_t(\delta_t, \theta_2; d_t, X_t, J_t)$ the vector of model-implied shares. Here the operator $f : \mathbb{R}^{J_t} \rightarrow \mathbb{R}^{J_t}$ for which we look a fixed point is given by:

$$f(\delta) = \delta + \log s_t - \log \tilde{s}_t(\delta, \theta_2; d_t, X_t, J_t)$$

On this operator we check the conditions for Berry's inversion. On the following we drop the t index and denote the model implied market share for asset j as $\tilde{s}_j := \tilde{s}_j(\delta, \theta_2; d_t, X_t, J_t)$.

- **Checking i.** We start by verifying that the first derivatives are non-negative, we have that:

$$\frac{\partial f_j(\delta)}{\partial \delta_k} = \begin{cases} 1 - \frac{1}{\tilde{s}_j} \frac{\partial \tilde{s}_j}{\partial \delta_j} & \text{if } k = j \\ -\frac{1}{\tilde{s}_j} \frac{\partial \tilde{s}_j}{\partial \delta_k} & \text{if } k \neq j \end{cases}$$

We know that $\tilde{s}_j \geq 0$ and using the definition for the model-implied shares we have that

$$\frac{\partial \tilde{s}_j}{\partial \delta_k} = \frac{\partial}{\partial \delta_k} \left[\sum_{i \in I} \left(\frac{A_i}{A} \right) \int w_{ij}(v_i) dF_v(v_i) \right] = \sum_{i \in I} \left(\frac{A_i}{A} \right) \int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i)$$

where $\frac{\partial w_{ij}(v_i)}{\partial \delta_k} = \begin{cases} w_{ij}(v_i)(1 - w_{ij}(v_i)) & \text{if } k = j \\ -w_{ik}(v_i)w_{ij}(v_i) & \text{if } k \neq j \end{cases}$

From the previous we see that if $k \neq j$ then $\frac{\partial w_{ij}}{\partial \delta_k} \leq 0$ and since $(A_i/A) > 0$ then $\frac{\partial \tilde{s}_j}{\partial \delta_k} \leq 0$. For $k = j$ notice that $\frac{\partial w_{ij}}{\partial \delta_k} \leq w_{ij}$ and then $\frac{\partial \tilde{s}_j}{\partial \delta_k} \leq \tilde{s}_j$ so $\frac{\partial \tilde{s}_j}{\partial \delta_k}$. The next step is to

verify that the sum of partial derivatives is less than 1. Notice that:

$$\begin{aligned}
\sum_{k=1}^{J_t} \frac{\partial f_j(\delta)}{\partial \delta_k} &= \frac{\partial f_j(\delta)}{\partial \delta_j} + \sum_{k \neq j} \frac{\partial f_j(\delta)}{\partial \delta_k} \\
&= 1 - \frac{1}{\tilde{s}_j} \frac{\partial \tilde{s}_j}{\partial \delta_j} + \sum_{k \neq j} \left(-\frac{1}{\tilde{s}_j} \frac{\partial \tilde{s}_j}{\partial \delta_k} \right) \\
&= 1 - \frac{1}{\tilde{s}_j} \left[\sum_{k=1}^{J_t} \frac{\partial \tilde{s}_j}{\partial \delta_k} \right] = 1 - \frac{1}{\tilde{s}_j} \left[\sum_{k=1}^{J_t} \sum_{i \in I} \left(\frac{A_i}{A} \right) \int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i) \right] \\
&= 1 - \frac{1}{\tilde{s}_j} \left[\sum_{i \in I} \left(\frac{A_i}{A} \right) \sum_{k=1}^{J_t} \int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i) \right]
\end{aligned}$$

Given the definition of \tilde{s}_j it is sufficient to show that $\sum_{k=1}^{J_t} \int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i) < \int w_{ij}(v_i) dF_v(v_i)$. For this we notice that

$$\begin{aligned}
\sum_{k=1}^{J_t} \frac{\partial w_{ij}(v_i)}{\partial \delta_k} &= \frac{\partial w_{ij}(v_i)}{\partial \delta_j} + \sum_{k \neq j} \frac{\partial w_{ij}(v_i)}{\partial \delta_k} = w_{ij}(v_i)(1 - w_{ij}(v_i)) - \sum_{k \neq j} w_{ij}(v_i)w_{ik}(v_i) \\
&= w_{ij}(v_i) \left[1 - \sum_{k \neq j} w_{ik}(v_i) \right] \leq w_{ij}(v_i)
\end{aligned}$$

- **Checking ii.** There first step is to rewrite the model-implied shares as

$$\begin{aligned}
\tilde{s}_j &= \sum_{i \in I} \left(\frac{A_i}{A} \right) \int w_{ij}(v_i) dF_v(v_i) = \exp(\delta_j) \sum_{i \in I} \left(\frac{A_i}{A} \right) D_{ij}(\delta) \\
\text{with } D_{ij}(\delta) &= \int \frac{\exp(h_{ij}(v_i))}{1 + \sum_{j=1}^J \exp(\delta_j + h_{ij}(v_i))} dF_v(v_i)
\end{aligned}$$

This implies that $\ln(\tilde{s}_j) = \delta_j + \ln(\sum_{i \in I} (A_i/A) D_{ij}(\delta))$ and that

$$f(\delta)_j = \ln(s_j) - \ln \left(\sum_{i \in I} (A_i/A) D_{ij}(\delta) \right)$$

Now notice that when $\delta_m \rightarrow -\infty$ for $m \neq j$ then the term $D_{ij}(\delta)$ tends to $\int \exp(h_{ij}(v_i)) dF_v(v_i)$

so a lower bound for $f(\delta)_j$ is

$$\underline{\delta}_j > \ln(s_j) - \ln \left[\sum_{i \in I} (A_i / A) \int \exp(h_{ij}(v_i)) dF_v(v_i) \right]$$

So condition ii. is satisfied with $\underline{\delta} := \min_j \underline{\delta}_j$.

- **Checking iii.** For this part set $\delta_k = -\infty$ for $k \neq j$ and defined $\bar{\delta}_j$ as the value of δ_j such that $\tilde{s}_0(\delta, \theta_2) = s_0$, that is the value of δ_j that along with $\delta_k = -\infty$ would match the observed shares for the outside good. Moreover let $\bar{\delta} > \max_j \bar{\delta}_j$.

If δ is such that $\exists j$ with $\delta_j > \bar{\delta}$ then $\tilde{s}_0(\delta) < s_0$ and hence $\sum_{k=1}^{I_t} \tilde{s}(\delta)_k > \sum_{k=1}^{I_t} s_k$ which means that there is a least one element k such that $\tilde{s}(\delta)_k > s_k$. For such k we have that $f(\delta)_k < \delta_k$ as required in part iii.

□

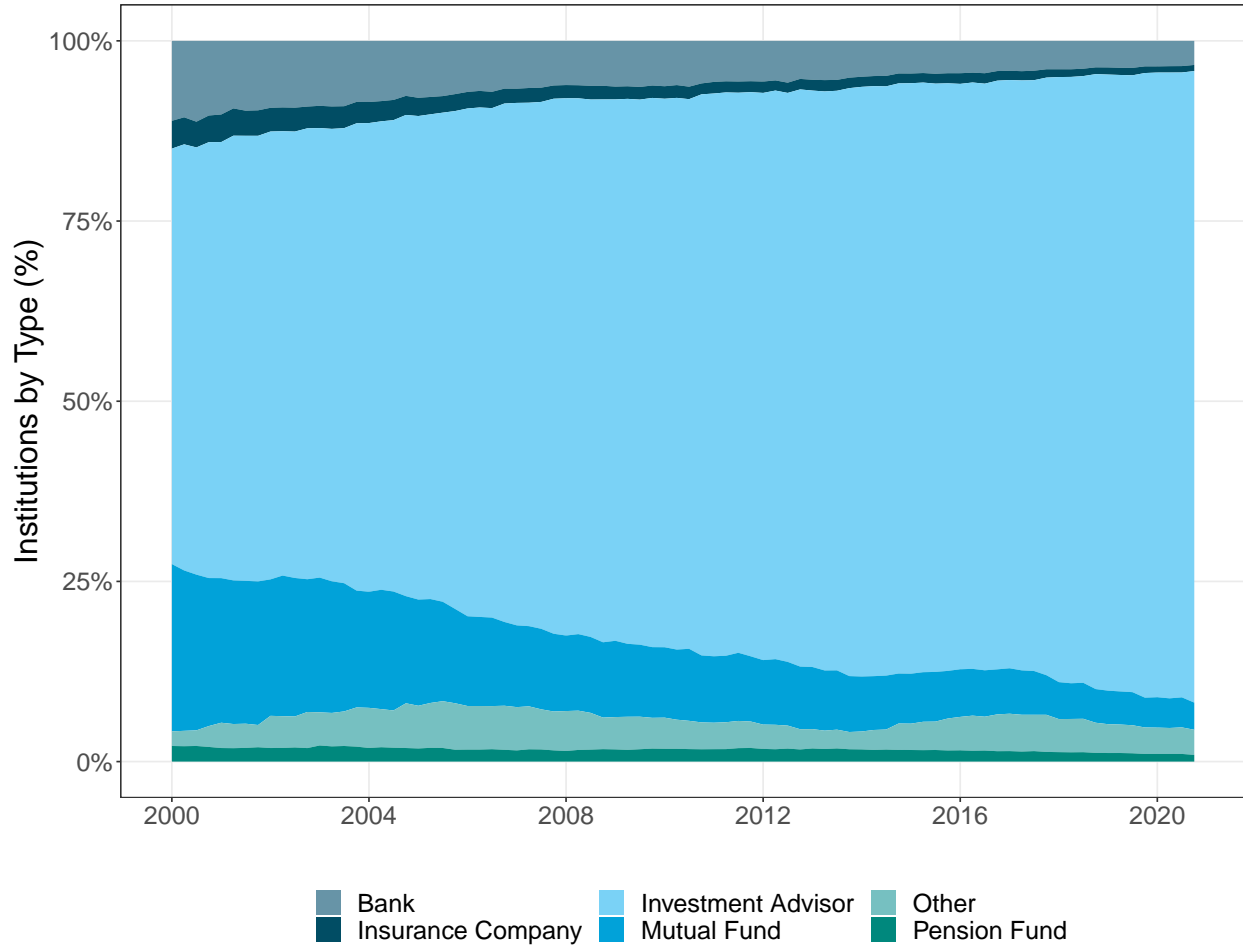
B Data Appendix

Table 2: Summary Statistics for Stock Characteristics

Variable	N. Stocks	Mean	Median	Std. Dev.	Min	Pc. 25th	Pc. 75th	Max
<u>2000-2004</u>								
Market Beta	499	0.684	0.639	0.554	-0.238	0.302	0.960	3.974
log Market Equity	499	8.009	7.985	1.900	0.598	6.897	9.175	13.256
log Total Assets	499	8.404	8.572	1.858	1.516	7.348	9.751	13.455
log Book-to-Market Equity	499	-0.408	-0.360	0.923	-5.521	-0.968	0.206	3.653
Profitability	499	0.211	0.192	0.227	-1.752	0.104	0.319	0.785
Investment	499	0.073	0.051	0.193	-0.811	-0.016	0.132	1.305
E-score	499	0.059	0.000	0.238	-0.500	-0.125	0.333	0.500
<u>2005-2009</u>								
Market Beta	696	1.208	1.099	0.713	-0.163	0.683	1.587	3.974
log Market Equity	696	8.038	7.981	1.804	0.923	6.884	9.299	13.149
log Total Assets	696	8.298	8.261	1.736	2.513	7.159	9.537	14.673
log Book-to-Market Equity	696	-0.482	-0.482	0.916	-5.659	-1.073	0.086	4.716
Profitability	696	0.223	0.211	0.251	-2.381	0.121	0.333	0.941
Investment	696	0.072	0.055	0.200	-0.741	-0.015	0.139	0.917
E-score	696	0.048	0.000	0.173	-0.500	0.000	0.200	0.500
<u>2010-2014</u>								
Market Beta	1122	1.294	1.223	0.646	0.009	0.812	1.678	3.182
log Market Equity	1122	8.460	8.558	1.801	1.169	7.345	9.657	13.374
log Total Assets	1122	8.665	8.650	1.869	1.534	7.539	9.871	14.697
log Book-to-Market Equity	1122	-0.599	-0.593	0.936	-7.699	-1.150	-0.002	4.218
Profitability	1122	0.227	0.209	0.241	-2.098	0.120	0.319	0.878
Investment	1122	0.062	0.046	0.151	-0.691	-0.007	0.114	0.816
E-score	1122	0.020	0.000	0.168	-0.500	0.000	0.125	0.500
<u>2015-2019</u>								
Market Beta	955	1.150	1.139	0.569	-0.396	0.779	1.487	3.184
log Market Equity	955	8.910	9.064	1.850	1.054	7.632	10.203	14.068
log Total Assets	955	9.003	8.949	1.903	1.534	7.803	10.263	14.780
log Book-to-Market Equity	955	-0.842	-0.794	1.061	-9.991	-1.441	-0.160	4.943
Profitability	955	0.261	0.223	0.292	-2.880	0.127	0.359	1.034
Investment	955	0.053	0.033	0.173	-0.633	-0.021	0.095	0.911
E-score	955	0.047	0.000	0.141	-0.500	0.000	0.125	0.500

Notes: Summary statistics for the stock's characteristics used during estimation. Statistics are computed over pooled quarterly observations of the variables every five years. Summary Statistics present mean, median, standard deviation, minimum, 25th percentile, 75th percentile and maximum.

Figure 6: Type of Institutional Investor



Notes: Evolution of institutional investors by type from 2000q1 to 2020q4. The classification of institutional investors follows the six categories as in Kojien and Yogo (2019).

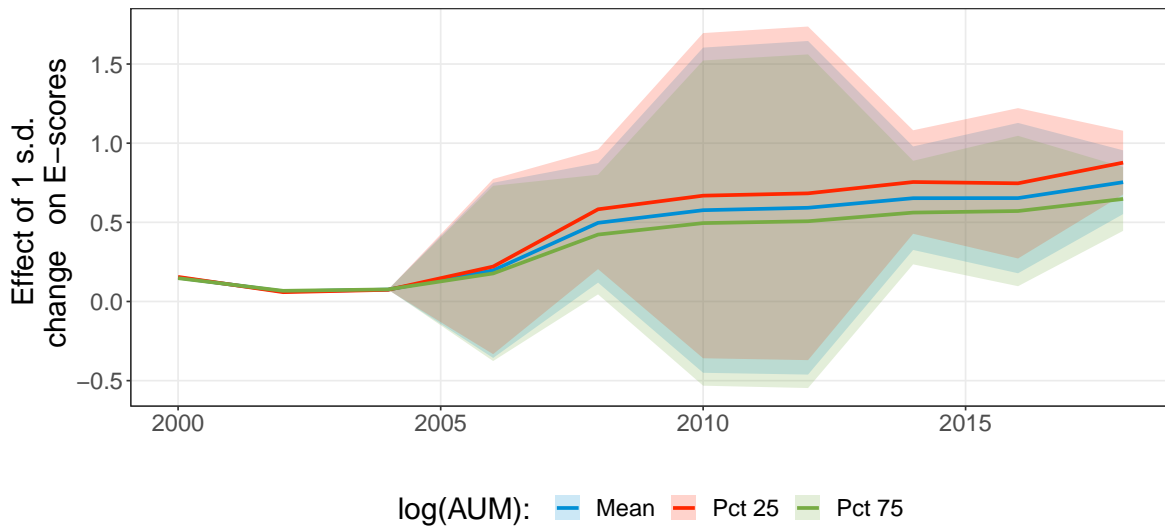
Table 3: Environmental Indicators from MSCI

Positive Indicators	Negative Indicators
Environmental Opportunities	Hazardous Waste
Waste Management	Regulatory Compliance
Packaging Materials and Waste	Ozone Depleting Chemicals
Climate Change	Toxic Spills and Releases
Environmental Management Systems	Agriculture Chemicals
Water Stress	Climate Change
Biodiversity and Land Use	Impact of Products and Services
Raw Material Sourcing	Biodiversity and Land Use
Natural Resource Use	Operational Waste
Environmental Opportunities - Green Buildings	Supply Chain Management
Environmental Opportunities in Renewable Energy	Water Management
Waste Management - Electronic Waste	Other Concerns
Climate Change - Product Carbon Footprint	
Climate Change - Insuring Climate Change Risk	
Other Strengths	

Notes: List of environmental performance indicators in the MSCI dataset. Each indicator is a dummy variable. The threshold for satisfying an indicator are determined by MSCI and are not disclosed with the data.

C Complementary Results

Figure 7: Estimated coefficients for E-scores



Notes: This plot shows the effective coefficient on E-scores, $\gamma_{it} = \gamma_0 + \kappa \log(\text{AUM})_{it} + \sigma v_{it}$, over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4. The plot uses various values, in each window, of log assets under management and shows the 95% confidence interval of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it} , are normally distributed.

Table 4: Counterfactual holdings and price changes of E-score-based portfolios

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
N. Stocks	104	104	104	103	103
ME (USD Bill.)	2847	2115	1814	5789	5356
Agg. Port. Share (%)	9.310	6.917	5.932	18.930	17.515
Counterfactual Agg. Port Shares (%)					
Logit	9.331	6.926	5.940	18.919	17.476
Mixed Logit	9.417	6.985	5.991	18.869	17.233
Counterfactual Price change (%)					
Logit	0.222	0.138	0.138	-0.059	-0.220
Mixed Logit	1.142	0.988	0.988	-0.319	-1.611
Price Change (%)					
2019 Q1 -2019-Q2	-1.046	3.496	2.606	2.950	4.734

Notes: This table shows the effect of a ban of green investing for pension funds on aggregate holdings and equity prices in a counterfactual exercise using data and estimates for 2019-Q1. The portfolios were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, while Portfolio 5 contains the 20% of stocks with highest E-scores. The table shows the counterfactual changes in aggregate portfolio holdings and value-weighted prices changes according to a logit demand specification and to a random coefficients (RC) demand specification. The observed value-weighted average price change between 2019-Q1 and 2019-Q2 for each portfolio is also shown.